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Non-Parametric Tests of Output- and Cost-Sharing Games

Spencer Banzhaf[1] and Yaqin Liu[2]

Abstract

The "tragedy of the commons" describes a variety of social dilemmas where total economic surplus is produced jointly from collective behavior and where individuals can strategically manipulate their share of the surplus. Recent research has shown that it is possible to test nonparametrically whether observed behavioral data are consistent with the canonical average return game, in which players share joint output in proportion to their inputs. We show that these tests extend to a much broader range of games, including equal-sharing of joint output, weighted averages of equal-sharing and proportionate sharing, and the average cost game, in which players share joint costs in proportion to the service provided them.

Key Words: Revealed Preference, Rationalization, Tragedy of the Commons, Common Pool Resources, Natural Resources.

JEL Codes: C14, D11, D21, Q20, Q30

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Non-Parametric Tests of Output- and Cost-Sharing Games

The "tragedy of the commons" (Hardin 1968) describes a variety of social dilemmas where total economic surplus is produced jointly from collective behavior and where individuals can strategically manipulate their share of the surplus. These surplus-sharing rules potentially create perverse incentive to provide too much effort in some cases (e.g., the race to fish) or shirk in others (e.g. a cooperative enterprise). Indeed, tragedy-of-the-commons games (with binary strategies) are N-person prisoners' dilemmas where each player can cooperate or defect, each benefits when more of the other players cooperate, and yet where defection is a dominant strategy.

However, it is not always clear whether tragedy-of-the-commons games are truly being played in a particular social setting (and hence whether the predicted Nash equilibrium is obtained), let alone which variant of the game. Needless to say, it would be useful to find behavioral tests of such behavior. Until recently, there have been few if any such tests of such behavior per se, only measures of aggregate outcomes. However, non-parametric revealed-preference-type tests have been extended to a variety of non-cooperative games, such as prisoners' dilemmas, without functional form assumptions about the underlying structure (Chambers and Echenique 2016). For example, Carvajal et al. (2013) showed how to non-parametrically test whether firms' behavior is consistent with Cournot behavior, another surplus-sharing game.

Recently, Banzhaf et al. (2024) showed that these approaches can be applied to one canonical commons model, often called the "average return game" (Gordon 1954, Sen 1966, Weitzman 1974, and Cornes and Sandler 1996). In this game, players choose how much effort (or other productive input) to apply to a joint production process, then receive a share of the joint output proportionate to their share of total effort. Examples include sending cattle to a common pasture (Hardin 1968), extracting groundwater (Ayres et al. 2021, Koch and Nax 2022), and fishing from the sea (Gordon 1954, Banzhaf et al. 2024). Koch and Nax (2022) have generalized these results to allow for a wider range of strategic complements and substitutes (e.g., groundwater use can affect a neighbor through positive or negative externalities).

In this short paper, we similarly extend the results of Banzhaf et al. (2024) to other social dilemmas involving common property. For example, Sen (1966) contrasted the average return game, with its sharing rule based on relative inputs, to an equal-sharing rule in a cooperative enterprise. Whereas the former results in excessive effort, the latter results in shirking. Additionally, Sen suggested the possibility of a sharing rule based on some average of these two, with the thought that the excessive inputs from the former and the insufficient inputs of the latter might cancel out and lead to an equilibrium closer to the optimum. We show that Sen's hybrid model can also be tested using nonparametric revealed-preference-like approaches. Furthermore, we show that data consistent with one of these models, including the equal-sharing game, will generally be inconsistent with another, thus giving power to the behavioral test.

Finally, we also extend our results to the so-called "average cost game," which is like a dual to the average-return game. Here, individuals choose their level of service and all pay proportionate shares of the joint cost. Examples include telephony, computer networks, and common facilities like clubs. Moulin and Watts (1997) compare the average cost and average return games, finding over utilization is more severe in the former.

1. The Average Return Game

As a point of departure, we first briefly review the results of Banzhaf et al. (2024) for the average return game. Consider an industry consisting of I profit-maximizing players, indexed by $i = 1$, 2, …, I, each having free access to an exogenously fixed common-pool resource. Depending on the application, these "players" could be firms, individual fishers, or individual workers on a collective farm, etc. There are T decision periods indexed by $t = 1, 2, \ldots, T$. Denote $q_{i,t}$ as the extraction effort by player i in period t. For example, $q_{i,t}$ might be the number of fishing vessel-days in year t. Let $Q_t = \sum_i q_{i,t}$ be the total level of effort applied to the resource at time t. Industry-wide revenue is a differentiable function $F_t(Q_t)$, with $F(0) = 0$, $F'(Q) > 0$, and F' non-increasing for all t. Note the revenue function $F_t(\,)$ varies by t, perhaps because of productivity shocks (e.g. weather affecting farming or fishing yields) and/or because of varying output prices. These are our only assumptions.

In the canonical commons model, player i's revenue is a share of total revenue proportionate to its share of effort or input. Thus, i's revenue in period t is $\frac{q_{i,t}}{Q_t} * F_t(Q_t)$. Finally, let $C_i(q_{i,t})$ denote i's cost of supplying effort, which is a differentiable convex function of q. We make no other assumptions beyond those stated thus far. Moulin and Watts (1997) show a Nash equilibrium always exists under these general conditions.

Consider player i's profit-maximization problem at time t:

(1) $$\max_{q_{i,t}} \frac{q_{i,t}}{Q_t} * F_t(Q_t) - C_i\,(q_{i,t}).$$

Taking others' actions as given, the first-order condition is:

(2) $$\frac{q_{i,t}}{Q_t} * F_t'(Q_t) + \left(1 - \frac{q_{i,t}}{Q_t}\right) * \frac{F_t(Q_t)}{Q_t} = C_{i,t}'.$$

This is the standard result that players equate marginal cost to a weighted average of marginal returns and average returns. In the case of a monopolist, $q_{i,t} = Q_t$ and the entire weight is on the efficient condition to equate marginal cost to marginal return. In the limit, as the number of players grows large, $q_{i,t}/Q_t$ goes to zero and they equate marginal cost to average revenue, thus depleting all resource rents.

Rearranging terms, we obtain:

(3) $$\frac{F_t(Q_t) - Q_t C_{i,t}'}{q_{i,t}} = \frac{F_t(Q_t)}{Q_t} - F_t'(Q_t).$$

Notice in Equation (3) that the left-hand side involves player-specific terms (inputs $q_{i,t}$ and marginal costs $C_{i,t}'$) while the right-hand side involves only market-wide data (total revenue $F_t(Q_t)$, marginal revenue product F_t', and total input Q_t). Consequently, from the first-order condition, to be consistent with the model the data must satisfy a "common ratio property:"

(4) $$\frac{\mathrm{F}_t(Q_t) - Q_t C_{i,t}'}{q_{i,t}} = \frac{\mathrm{F}_t(Q_t) - Q_t C_{j,t}'}{q_{j,t}} \geq 0 \;\; \forall\, i, j, \forall\, t\,.$$

That is, in each period, functions of the total extraction effort and player-specific marginal costs should all be equal. The expressions are nonnegative given the concavity of the production function.

Moreover, because each player's cost function is convex, under the model the data also must satisfy the co-monotone property, such that for all i,

(5) $$\left(q_{i,t} - q_{i,t'}\right)\left(C_{i,t}' - C_{i,t'}'\right) \geq 0 \;\; \forall\, i, \forall\, t, t'\,.$$

Consequently, a set of observations is consistent with the tragedy of the commons with convex cost functions if and only if there exist nonnegative numbers $\{C'_{i,t}\}$ for all i, t that obey the common ratio and co-monotone properties. That is, we can test for whether behavior is consistent with the average-return game without making any functional form assumptions about $F_t(\,)$ or $C(\,)$, beyond concavity and convexity respectively plus differentiability.

Banzhaf et al.'s approach to testing whether behavior is consistent with this model can be formulated as a simple linear program. A set of observations is consistent with the tragedy of the commons with convex cost functions if and only if, given the observed F_t, $q_{i,t}$, and Q_t, there are non-negative numbers $C'_{i,t}$ satisfying Conditions (4) and (5) $\forall\, i, t$.

2. Cooperative Enterprises

Sen (1966) considered an application of the average return game to cooperative farms and other enterprises, where a planner measures workers' labor input and divides output according to the above proportionate sharing rule. He also contrasted this proportionate sharing rule with a rule where each worker gets an equal share. According to Sen, the former follows Karl Marx's dictum of "each according to his work," whereas the latter follows his dictum of "each according to his needs" (assuming everybody has equal needs).

Under the equal sharing rule, each worker's objective is:

$$(6) \qquad \max_{q_{i,t}} \ \frac{1}{I} F_t(Q_t) - C_i\,(q_{i,t}).$$

Taking other workers' actions as given, *i*'s first-order condition is:

$$(7) \qquad \frac{1}{I} F'_t(Q_t) = C'_{i,t}.$$

In this case, workers set only a fraction $1/I$ of the marginal return equal to the marginal cost of effort, instead of the full marginal return. That is, in contrast to the average return game, where they had an incentive to over-supply effort, now they have an incentive to shirk, or free ride.

Note that, according to the first-order condition (7), in any time period t, $C'_{i,t}$ must be the same for all *i*:

(8) $$C'_{i,t} = C'_{j,t} \; \forall \, i, j, \forall \, t.$$

It follows that a system is consistent with the equal sharing rule if and only if we can find non-negative numbers $C'_{i,t}$ satisfying Conditions (5) and (8) $\forall \, i, j \in I, \forall \, t \in T$. Unfortunately, given the weak inequality in Condition (5), these conditions can be satisfied trivially by choosing any positive marginal costs. This is consistent with other limiting cases in the literature. For example, in the Cournet model, the limiting case of perfect competition also can never be rejected, for similar reasons (Carvajal et el. 2013).

However, we can still differentiate this case from the average return game. In general, any non-negative numbers $C'_{i,t}$ satisfying both Conditions (4) and (5) will not necessarily also satisfy Condition (8) (or vice versa). Condition (8) requires all the $C'_{i,t}$ to be the same within *t*, whereas Condition (4) requires a common ratio based on the observed $q_{i,t}$. Thus, a dataset can be consistent with one model but not the other, making the tests economically meaningful.[1]

3. Hybrid Sharing Rules

Moreover, as we move away from equal sharing as one polar case, the test again gains power. Sen (1966) also considered hybrid sharing rules, where a proportion of output α is distributed equally ("according to needs") and (1-α) is distributed proportionate with effort ("according to work"). Sen showed that there is an optimal α* that offsets the problem of each, shirking under equal distribution and over-extraction under proportionate sharing. (See also Heintzelman et al. 2009).

In this case, the objective function is:

(9) $$\max_{q_{i,t}} \; \alpha \frac{1}{I} F_t(Q_t) + (1-\alpha) \frac{q_{i,t}}{Q_t} F_t(Q_t) - C_i\,(q_{i,t}).$$

The first order condition is:

(10) $$\alpha \frac{F'_t(Q_t)}{I} + (1-\alpha) \left[\frac{q_{i,t}}{Q_t} F'_t(Q_t) + \left(1 - \frac{q_{i,t}}{Q_t}\right) * \frac{F_t(Q_t)}{Q_t} \right] = C'_{i,t},$$

[1] Of course, like any test, it is not *always* decisive. There could be some data sets where it is possible to find one set of non-negative $C'_{i,t}$ that satisfy Conditions (4) and (5) and a different set of non-negative $C'_{i,t}$ that satisfy Conditions (5) and (8).

which can be re-arranged as:

(11) $$\frac{C'_{i,t}}{k_{i,t}} - \frac{1-\alpha}{k_{i,t}}\left(1 - \frac{q_{i,t}}{Q_t}\right)\frac{F_t(Q_t)}{Q_t} = F'_t(Q_t),$$

where $k_{i,t} = \frac{\alpha}{I} + (1-\alpha)\frac{q_{i,t}}{Q_t}$. Again, all player-specific information is on the left, so the left hand side of Equation (11) must be the same for all players:

(12) $$\frac{C'_{i,t}}{k_{i,t}} - \frac{1-\alpha}{k_{i,t}}\left(1 - \frac{q_{i,t}}{Q_t}\right)\frac{F_t(Q_t)}{Q_t} = \frac{C'_{j,t}}{k_{j,t}} - \frac{1-\alpha}{k_{j,t}}\left(1 - \frac{q_{j,t}}{Q_t}\right)\frac{F_t(Q_t)}{Q_t}, \ \forall\, i, j, \forall\, t.$$

And likewise, the test again comes down to finding non-negative numbers $C'_{i,t}$ that satisfy Conditions (5) and (12) $\forall\, i, j, \forall\, t$. In this case, the expression is somewhat more complicated, but note that the other terms involving k are simply arithmetic functions of the data and of either a known α or arbitrary α which defines the game to be tested. It can easily be verified that for α equal to zero or one, the expression collapses to those above.

4. The Average Cost Game

So far, in all the games we have considered, players choose inputs and receive outputs based on sharing rules. Consider now the opposite case, where players sharing a resource choose to receive a level of services (output) $y_{i,t}$, with the total output being Y_t. Each player has an individual-specific concave value function, $u_i(y_{i,t})$. $C_t(Y_t)$ is the cost of the total service level and each player's cost is proportional to his/her level of services. For example, total costs of a golf club are shared out on a per-round basis. Thus i's problem is:

(13) $$\max_{y_{i,t}}\left\{u_i(y_{i,t}) - \frac{y_{i,t}}{Y_t} * C_t(Y_t)\right\}$$

The first-order condition is

(14) $$\frac{y_{i,t}}{Y_t} * C'_t(Y_t) - \frac{y_{i,t}}{{Y_t}^2} * C_t(Y_t) + \frac{C_t(Y_t)}{Y_t} = u'_{i,t}.$$

Rearranging terms we get:

(15) $\frac{C_t(Y_t)-Y_t u'_{i,t}}{y_{i,t}} = \frac{C_t(Y_t)}{Y_t} - C'_t(Y_t).$

This expression is analogous to equation (3) with $F_t(Q_t)$ replaced by the total cost $C_t(Y_t)$ and $C'_{i,t}$ is replaced by individual utility $u'_{i,t}$. From Equation (15) we obtain the common ratio property:

(16) $$\frac{C_t(Y_t) - Y_t u'_{i,t}}{y_{i,t}} = \frac{C_t(Y_t) - Y_t u'_{j,t}}{y_{j,t}} \leq 0\,, \qquad \forall\, i, j, \forall\, t.$$

Notice that the common ratio is now negative given the convexity of the cost function.

Likewise, with concave *u*( ), the co-monotone property remains analogous but with the sign reversed:

(17) $\left(y_{i,t} - y_{i,t'}\right)\left(u'_{i,t} - u'_{i,t'}\right) \leq 0 \;\; \forall\, i, \forall\, t, t',$

Thus, a panel data is consistent with the average-cost game if there exist non-negative numbers $\{u'_{i,t}\}$ consistent with Conditions (16) and (17).

5. Conclusion

There are a wide variety of settings where people share common resources and divide outputs or costs according to different institutional rules. We have shown that it is possible to test whether individual behavior is consistent with classical examples of these institutions.

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