Optimal Management of Spread Externalities and the Implications of Heterogeneous Capacity Constraints

James S. Goodenberger[1]

Sathya Gopalakrishnan[2]

H. Allen Klaiber[2]

[1] Department of Economics
Washington and Lee University

[2] Department of Agricultural, Environmental and Development Economics
The Ohio State University

Abstract

Ecological conditions give rise to heterogeneous carrying capacities that influence both the growth and spread of spatial externalities. We develop a spatial-dynamic model of resource management and show that ecological capacity constraints influence optimal control strategies across space. We use integer-programming methods to solve our model for optimal control strategies in both homogeneous and heterogeneous landscapes. Our results indicate that when spatially heterogeneous capacity constraints are included in the model, the optimal levels of suppression will vary over space as well. This occurs for two reasons. The first is that heterogeneous capacity constraints influence the costs and damages associated with the externality, therefore, discrete patches with different carrying capacities have different optimal levels of control. The second reason is that spread externalities exist and become more prevalent as the heterogeneity between adjacent patches increases. We show that ignoring spatial heterogeneity leads to welfare losses. In numerical models with high degrees of heterogeneity, we show that a naïve homogeneous carrying-capacity assumption results in significant increases in the total cost of control. This highlights the need to account for and model heterogeneous carrying capacities in the optimal management of spatial-dynamic externalities.

Keywords: invasive species; optimal control, spatial-dynamic externalities; spatial heterogeneity, Asian carp

JEL Codes: Q20, Q22

Optimal Management of Spread Externalities and the Implications of Heterogeneous Capacity Constraints

1 Introduction

Understanding the welfare implications of economic decisions in the presence of externalities is foundational to environmental and resource economics. Managing and regulating externalities that evolve over time and simultaneously spread across space, however, presents a challenging problem in which economic decisions and natural resource stocks are linked through spatial-dynamic processes (Smith et al. 2009). To manage these externalities, such as the spread of invasive species, wild fires, or infectious diseases, policymakers must determine both when and where to invest in control effort. Whereas individual components of space and time influencing these decisions have been widely studied by resource economists, research on spatial-temporal links is relatively nascent. We extend this growing literature by developing a spatial-dynamic model for optimal management that simultaneously considers heterogeneity in the carrying capacity of resource stocks, stock intensities, and economic damages across space.

Modeling the management of spread externalities presents tradeoffs between computational tractability and realistic representations of ecological and economic characteristics that influence policy outcomes. Spatial-dynamic models of renewable resource management are growing in the literature (Sanchirico and Wilen 1999, Costello and Polasky 2008, Brock and Xepapadaes 2010, Gopalakrishnan et al. 2017), and typically involve resource stocks that are governed by physical or biological processes following diffusion or dispersal of the stock. Examples of resources that involve spatial-dynamic processes include the spread of an invasive species (Epanchin-Niel and Wilen 2012; Fenichel et al. 2014), the diffusion of fish biomass (Sanchirico and Wilen 2005), and the transmission of infectious diseases. An important distinction between spatial effects and spatial-dynamic effects is that the externality, or public bad, through the evolution of resource stocks across location, makes spatial outcomes endogenous. Analytical solutions in these models are often challenging because determining the optimal policy often involves solving a complex system of partial differential equations (Brock and Xepapadeas 2008), and optimal outcomes depend on spatial geometry of the landscape (Epanchin-Niel and Wilen 2012; Smith et al 2009). Numerical solution methods can accommodate complex characterizations of the landscape and determine both optimal management paths and steady state outcomes (Epanchin-Niel and Wilen 2012).

The literature has largely focused on socially optimal management of negative externalities to minimize damages and costs (Archer and Shogren 1996; Olson and Roy 2002). While previous work addresses the question of *when* it is optimal to control (Homans and Horie 2011; Leung et al. 2002; Burnett et al. 2006), it ignores complex spatial dimensions of real-world policy. Studies that have addressed spatial concerns and strategic interactions between neighboring private owners of the resource are often limited to two distinct spatial regions for analytical tractability (Grimsrud et al 2008; Fenichel et al 2014; Epanchin-Neill and Wilen 2015). Recent analytical work on spread externalities in a spatially extended landscape, examines conditions under which outcomes of non-cooperative private resource ownership generate socially optimal outcomes (Costello et al. 2017).

In this paper, we develop a spatial-dynamic model of resource management in a landscape that incorporates heterogeneity in ecological characteristics (carrying capacity), and in the intensity of the stock externality. Heterogeneity in carrying capacity influences the intensity as well as spread of the negative externality. Intensity and ecological carrying capacities also affect control costs and the extent of damages. In our model, spread dynamics depend on the gradient in stock intensities over space. That is, if left untreated, damages from the externality spread from a location with higher intensity to a neighboring location with lower intensity. This mimics real-world diffusion processes, and introduces spatial externalities if a social planner fails to recognize the potential for increased spread to neighboring patches, and associated control and damage costs of leaving a high intensity patch unmanaged.

This paper contributes to the existing literature in three ways. First, we examine the role of heterogeneity in ecological carrying capacities in modeling the optimal control of spatial externalities. Second, we consider the tradeoff between localized management in a single location and minimizing spread damages in the presence of spatial-dynamic externalities. We show that optimal tolerance for spread externality varies in a complex ecological landscape, and ignoring heterogeneity in ecological constraints leads to welfare loss. Furthermore, the welfare loss increases with the magnitude of variability in carrying capacity within a landscape. Finally, we demonstrate the importance of incorporating heterogeneity when economic conditions and carrying capacities are endogenous.

In the following section, we develop a discrete-space, discrete-time model in which a social planner minimizes the discounted sum of damages and control costs in a landscape with ecological constraints. We numerically solve the model using integer programming and examine the role of stock intensity and spread externalities on optimal control strategies in a spatially heterogeneous landscape. In Section 3 we discuss results and compare outcomes across homogenous and heterogeneous landscapes. We also demonstrate the implications of an externality spreading from a rural area into a more populated area, like a forest bordering a town. We conclude with a discussion of policy implications in the final section.

2. Discrete Dynamic Model of Invasive Control

We develop a discrete-space, discrete-time model of optimal resource management in a complex landscape with a spread externality. The general model representing a social planner's problem can be written as:

$$\min_{s_{it}} \left\{ \sum_t \sum_i \beta^t \big(D(x_{it}; k_i) + C(s_{it}, x_{it}; k_i) \big) \right\} \tag{1}$$

Subject to:

$$x_{i(t+1)} = x_{it} + \max\Big(F(x_{it}; k_i), G\big(x_{(i-1)t}, x_{(i+1)t}; k_i\big) \Big) - s_{it} \tag{2}$$

In equations (1-2) the state variable, x_{it}, represents the intensity of externality in location i at time t. s_{it} is a control variable that represents economic behavior – decisions to control the externality through clean-up or suppression mechanisms, t ∈ [0, T] indexes time and $i \in [1, N]$ indexes space where N is the number of discrete patches. β is the discount factor. Each location i is influenced by exogenous ecological processes that determine its carrying capacity, k_i. $D(x_{it}; k_i)$ is the instantaneous damage caused by the externality and the cost function, $C(s_t, x_{it}; k_i)$, represents control costs. The function $F(x_{it}; k_i)$ represents the evolution of the stock externality within patch i that depends on the intensity of damage at time t, x_{it}, and the carrying capacity in each patch, k_i. The function $G\big(x_{(i-1)t}, x_{(i+1)t}; k_i\big)$ represents spread from adjacent patches $(i - 1)$ and $(i + 1)$ into patch i. The social planner's objective is to choose actions simultaneously over time and across space to minimize the stream of discounted damages and control costs.

Growth in the externality stock is linear within a patch. In the absence of any control, intensity of damage in patch i increases by 1 unit in each time step until it reaches the carrying capacity. Therefore, $F(x_{it}; k_i) = 1 \; \forall \; 0 < x_{it} < k_i; 0$ otherwise. We model spatial-dynamic interaction across patches through a gradient-dependent spread such that the difference in intensity between adjacent patches determines spread of the externality from higher to the lower intensity patches. $G\left(x_{(i-1)t}, x_{(i+1)t}; k_i\right) = \max\{0, \left(x_{(i-1)t} - x_{it}\right), \left(x_{(i+1)t} - x_{it}\right)\}$.[1]

In the absence of any control following an exogenous introduction in the initial time period (t=0), the externality-producing stock grows within each invaded location and spreads to adjacent patches. For example, consider a landscape of 7 patches and carrying capacities, k_i, for each patch represented by the vector $K = [3,1,4,5,2,2,5]$. In time period zero, the location of initial damage is discovered by the social planner, $x_{4,0} = 1$. In subsequent time periods intensity grows by 1 unit, and the damage spreads from patches with higher intensities to adjacent patches with lower intensities. This spatial-dynamic growth process is constrained by carrying capacity as illustrated in Figure 1. Zero-flux boundary conditions are assumed, which is equivalent to assuming a carrying capacity of zero outside of the spatial domain. [2]

The social planner then chooses the timing, location, and quantity of control. It is possible to suppress intensity to any non-negative quantity, but the cost of suppression increases as the level of intensity decreases due to increased search costs associated with more limited stock. To solve the model using integer programming, the state and control variables are constrained to be integers; $x_{it} \in [0, k_i]$, $s_{it} \in [0, x_{it}]$.

2.1. **Damage and Control Cost Functions**

Quantifying the damages imposed by an externality is critical for determining optimal control, since damages avoided measure the benefits from management. In this model, marginal damages depend on three landscape characteristics: carrying capacity, intensity of externality-

[1] Results are qualitatively similar in a model with a density-dependent diffusion process for the spread of invasive populations (See Appendix B). However, computational complexity limits the spatial extent of the landscape.
[2] Results are qualitatively similar with a circular boundary condition.

producing stock, and a baseline value of the patch (which could reflect human population densities). The marginal damage associated with patch i being affected in time period t is given by:

$$D(x_{it}; k_i) = p_i * \frac{x_{it}}{k_i} \quad (3)$$

where p_i is the baseline value of patch i. Damages are increasing in baseline patch value $\left(\frac{\partial D}{\partial p_i} \geq 0\right)$ and in intensity of the externality-producing stock $\left(\frac{\partial D}{\partial x_{it}} \geq 0\right)$ but are decreasing in carrying capacity $\left(\frac{\partial D}{\partial k_i} \leq 0\right)$, which reflects lower competition for resources in locations with higher carrying capacity.

Suppression costs depend on the degree of suppression employed, s_{it}, the carrying capacity of the patch, k_i, and the intensity of resource stock that produces the externality, x_{it}. Suppression is a control strategy that reduces the intensity or prevents growth of the damaging stock within a patch. As the intensity of a patch decreases relative to its carrying capacity the amount of effort required to find and suppress additional units increases. Therefore, similar to a marginal abatement costs in pollution control, the marginal suppression cost function is increasing in k_j and decreasing in x_i. Eradicating the spread externality is very difficult and imposes considerably higher costs than suppression. To reflect this additional cost, the cost function is specified piecewise at the point where eradication occurs. α and γ are suppression and eradication cost parameters. The marginal cost of suppression therefore depends on the carrying capacity and the extent to which suppression controls the stock within a location:

$$C(s_{it}) = \alpha * \frac{k_i}{(x_{it} - s_{it})} \qquad for\ s_{it} < x_{it} \quad (4a)$$

When an eradication control is implemented, the costs are given by:

$$C(s_{it}) = \gamma * k_i \qquad for\ s_{it} = x_{it} \quad (4b)$$

2.2. Numerical Solution Method

We solve the discrete time, discrete space optimal management problem as a binary integer-programming problem. Integer constrained optimization problems of this type have a large set of feasible solutions, and are analytically intractable. Consider, for example, a grid-space of 10x25 (25 patches with uniform carrying capacity of 10) that is solved over a 150 period time horizon. In this scenario, there are 37,500 binary state variables and 37,500 binary control variables, meaning that the solution set would contain $2^{(75,000)}$ possible solutions. To tackle a problem with such a large solution set we employ the numerical 'branch and bound' method (Land and Doig 1960) and solve the model in MATLAB. In a binary integer model, the Land and Doig method first ignores the integer constraints and solves the resulting linear programming problem. If the solution is composed entirely of integers, then the problem is solved. In the majority of problems, however, solutions are not composed entirely of integers making this solution process more challenging. Typically, some, or all, of the elements in the optimal solution vector are non-integers. In this case, the problem is split into two new problems by selecting one of the non-integer valued variables and creating a problem where this variable is assumed equal to zero and another where this variable is assumed to be equal to one. This process is repeated on each of the resulting problems, stopping only when an all integer solution is found. The value function is evaluated at the solution and compared with all other active nodes in the solution 'tree'. All strictly dominated nodes are eliminated or 'pruned', thereby, eliminating portions of the solution set and reducing the computational intensity of the problem. This solution method yields a globally optimal solution to the binary integer-programming problem as the entire potential solution space is covered.[3]

3. Optimal management solutions

To analyze the impact of carrying capacities on optimal control strategies, we first consider long-run outcomes in homogeneous landscapes. Steady-state solutions in a homogeneous landscape serve as a baseline for comparison with optimal control strategies in heterogeneous environments. We use these comparisons to examine inefficiencies in 'naïve' management strategies, where the central planner fails to account for the spatially heterogeneous nature of the landscape, or assumes the landscape homogeneous when making control decisions.

3.1. Homogeneous Carrying Capacity

First, we consider the base scenario, with a single, low-intensity stock that produces a negative externality, in a landscape divided into 25 patches with uniform carrying capacities. In a homogeneous landscape, with uniform costs and damages across space, optimal steady states will also be homogenous across space. Three types of steady state outcomes are possible for every scenario with homogenous carrying capacity landscapes: no management, immediate eradication, or and interior solution with suppression to a uniform intensity level.[4] The realization of one of these three outcomes depends on several factors including the carrying capacity and size of the landscape, the location of the initial stock, and the relative magnitudes of damages and control costs. Here, we examine the effect of carrying capacity constraints on the longterm outcomes.

Carrying capacity in a homogeneous landscape can influence the optimal control strategy in two ways. First, it determines the type of control that is optimal (i.e. immediate eradication vs

[3] The long-run solution may not be global in the case of discontinuous or stochastic propagation of stock over space. In this setting, elimination of dominated nodes is not sufficient to ensure a globally optimal solution.
[4] An interior solution in homogeneous landscapes does not exist when the suppression cost function is constant as shown in Appendix A.1.

suppression vs no management). An example is shown in figure 2, where two landscapes differing only in carrying capacity are compared (Figure 2a: $k_i = 10$, Figure 2b: $k_i = 15$). In figure 2a, smaller carrying capacities make immediate eradication relatively cheaper than suppressing a large area over a long time horizon. However, the opposite is true in the high carrying capacity landscape in figure 2b. In this landscape, eradication is costlier; therefore, suppression across the landscape is optimal.

The second way that carrying capacity can influence the optimal control strategy is by altering the timing of suppression and thus the optimal intensity of the invasion. This is shown in figure 3 where each of the three landscapes have a different carrying capacity (Figure 3a: $k_i = 5$, Figure 3b: $k_i = 10$, Figure 3c: $k_i = 15$). Note that though the optimal level of the externality-producing stock is different in each landscape (2, 4, and 6 respectively), the relative tolerance is 40% of the total carrying capacity in each landscape. When suppression is optimal in homogeneous landscapes, the optimal intensity may be some constant proportion of the total carrying capacity.

These results show that carrying capacity plays a role in determining the optimal control strategies and intensity levels in homogeneous landscapes. However, due to the assumption of homogeneity, optimal control strategies are homogeneous across space as well. We will now examine optimal management when carrying capacities are spatially heterogeneous.

3.2. Heterogeneous Carrying Capacity

Landscapes are rarely homogeneous in the real world. Geophysical features that influence the spatial geometry of the landscape, and ecological constraints, such as natural fluctuations in food supply, vegetation, or competing populations, can alter the ecosystem capacity to sustain any resource stock. Therefore, to increase ecological realism we introduce heterogeneous ecological constraints across space. As in the homogeneous case discussed above, landscapes are divided into 25 discrete patches but the carrying capacity in each patch is now drawn from a normal distribution centered at 10. Results are therefore comparable to similar landscapes discussed in section 3.1.

Again, immediate eradication, no management, and suppression are all feasible solutions. However, carrying capacity has an larger effect on optimal management strategies when spatial heterogeneity exists. Unlike outcomes in homogeneous landscapes, an interior solution results in non-uniform stock levels across space (see figure 4). When the carrying capacity in a patch is higher, damages from any given level of the externality-producing stock are relatively cheaper than the cost of suppression or control. Thus, the central planner allows the stock to grow before investing in control in locations with high carrying capacity. Optimal steady-state intensity levels vary throughout the landscape in a pattern that reflects both variation in carrying capacities and gradients across locations. If individual carrying capacities were the sole driver of the optimal intensity of invasion, then all patches with identical carrying capacities would be suppressed at the same level. This, however, is not the case, as can be observed by comparing patch 2 with patch 6. Both have 13 units of carrying capacity, yet the optimal level of intensity for patch 2 is 5 while patch 6 has an optimal intensity of 4. This result highlights the effect of spatial spread externalities on optimal control strategies and demonstrates tradeoffs between localized damage control in each location and minimizing damages from spread externalities by reducing the stock gradient across locations.

3.3 Heterogeneous Landscapes with Heterogeneous Patch Values

We now turn to a scenario in which carrying capacities are altered to varying degrees by human activities, such as the construction of homes. This results in a landscape with a high carrying capacity and low valued area on the left, which gradually transitions into a low carrying capacity and highly valued area on the right. While the general trend is one of decreasing carrying capacity from left to right, some natural variation is also incorporated. With increased human presence within a patch, any given level of negative externality producing stock causes relatively more damage than in a non-inhabited patch. This is reflected in the model through the baseline patch value parameter, p_i, which increases from 1 to 13 by 0.5 increments from left to right.

Figure 5a shows the optimal control steady state when the central planner acknowledges the heterogeneity of the landscape. Figure 5b shows the optimal control steady state if the central planner chose to ignore heterogeneity and instead assume that each patch had a carrying capacity equal to the mean carrying capacity of the landscape, which is 10. In both cases, the central planner acknowledges the differences in baseline patch values that exist across the landscape.

Interestingly, when heterogeneous carrying capacities are ignored the central planner will hold the stock intensity to a sub-optimally low intensity level in the more remote portion of the landscape (patches 1-3, 6, and 8) while allowing the stock intensity to be sub-optimally high in some of the areas with more human activity (patches 15-18). This spatial misallocation of resources is a direct consequence of ignoring the spatial heterogeneity in carrying capacities within the landscape.

3.3. Inefficiency of a Naïve Central Planner

In this section, we examine inefficiencies that result from the central planner's naïve strategy of ignoring spatial heterogeneity in carrying capacities. We demonstrate that welfare costs grow as the variance in carrying capacity increases.

Previously, we have shown that homogeneous landscapes have optimal control strategies that do not vary over space. Therefore, if a central planner were to treat a heterogeneous landscape as if it were homogeneous, they would pursue the control strategy shown in figure 6b instead of the optimal strategy in figure 6a. The uniform suppression strategy is sub-optimal because it results in over-investment in some locations and under-investment in others. As a result, the total present value of damages and control costs are higher than in the cost-minimizing solution. In other scenarios, the inefficiency may stem from pursuing the incorrect type of control all together, such as suppression instead of immediate eradication.

Figure 7 shows that the magnitude of inefficiency depends on the variability of carrying capacities in the landscape. We simulated 250 landscapes, with carrying capacities drawn from 5 separate normal distributions centered around 10. The standard deviations of each normal distribution differed from one to five (50 landscapes were drawn from each distribution). We solved for the optimal control strategy in each landscape and evaluated the value function, for a range cost and damage parameters. We then calculated inefficiency costs by comparing the value functions associated with optimal control paths and the sub-optimal strategy of treating each landscape as homogeneous. The average inefficiencies are presented in figure 7 with the standard deviation of the carrying capacities on the x-axis and the average cost inefficiency, as a percentage, reflected on the y-axis. The thick black line, which separates the bottom-left from the top-right, divides the parameter space by the type of management strategy chosen in a homogeneous landscape. For combinations of cost and damage parameters to the left of the thick black line, the assumption of homogeneity results in immediate eradication. For parameterizations to the right, the assumption of homogeneity results in suppression to an interior intensity level. We find that inefficiencies are largest for the parameterizations close to this separatrix. In other words, inefficiencies are larger on average when the homogeneity assumption is most likely to cause the central planner to choose the incorrect type of control. In Figure 8, we show a contour plot that reflects this result, clearly indicating that inefficiency costs increase in regions close to the separatrix.

The probability that the central planner's control method will be incorrect is worsened by the fact that the inefficiencies of choosing the incorrect type of control are much higher than those that occur when the correct type of control is chosen. This is shown in figures 9 and 10. Figure 9 shows average inefficiencies when eradication is the optimal strategy. Obviously, eradication leads to no inefficiencies when eradication is optimal, but when suppression occurs instead, large inefficiencies arise with averages rising near 25% in landscapes with high carrying capacity variances. When suppression is optimal, inefficiencies exist regardless of the chosen control method. If the central planner chooses to eradicate, the immediate high costs exceed the present and future discounted damages and costs from the optimal suppression strategy. This leads to higher inefficiencies between 10% and 20% in landscapes with highly variable carrying capacities. Even when the central planner chooses to suppress, a uniform suppression strategy will likely be sub-optimal.

4. Discussion

We develop a spatial-dynamic model of resource management in the presence of spread externalities and show that heterogeneous ecological constraints or carrying capacities influence optimal control strategies across space. Our model highlights tradeoffs between controlling economic damages in a single location and minimizing spatial externalities from the spread of damages across space, and provides new insights for the management. Our results complement existing work in spatial-dynamic management of renewable resources (Costello et al. 2017; Sanchirico et al 2010, Smith et al 2009, Epanchin-Neill and Wilen 2012, Sanchirico and Wilen

1999) and highlight the need for spatially targeted policies to optimally control spatial dynamic externalities. This work leads to an important policy insight demonstrating that management of renewable resources with spatial-dynamic externalities depend not only on spatial geometry and boundary conditions (Epanchin-Neill and Wilen 2012; Smith et al. 2009), but also on heterogeneous ecological constraints within these complex landscapes.

Our results provide new insights to planners, and underscore the need to recognize ecological carrying capacities and thresholds when managing resource stocks in patchy environments. When such heterogeneity is ignored, sub-optimal management policies – in the type of control and timing of control – can result in significant inefficiency costs that increase with higher variability in the landscape. In this analysis we focus on optimal control strategies when capacity constraints are exogenously determined as well as when ecological patchiness is caused by human activity. Although modeling land use patterns that influence ecological carrying capacities over longer time scales is beyond the scope of this analysis, our model can inform policy in the short run to guide transitions in land use patterns.

We find that it is optimal for a central planner to minimize the variation of intensities between neighboring patches. Because spread externalities result in increasing damages and control costs, optimal suppression activity should be implemented not only in high value areas, but also in the patches surrounding it. This ensures that invasive populations do not build in areas near those being controlled and that the control policy is cost effective.

This numerical model has a wide range of applications such as invasive species, forest fires, epidemics, or other processes with spatial-dynamic externalities where heterogeneous stock intensity across space is a central concern for policymakers. This work offers a key policy insight by demonstrating that, in complex ecosystems, cost-minimizing policies may require non-uniform investments across space. However, optimal patterns of resource management will likely not be achieved when decisions are made locally without regard to the spatial implications. While policy challenges in designing and implementing heterogeneous control require further research, our model offers a first step towards determining optimal transition paths and the appropriate jurisdictional scope, which depends on landscape features and ecosystem constraints.

References

Albers, Heidi J., Carolyn Fischer, and James N. Sanchirico. “Invasive species management in a spatially heterogeneous world: Effects of uniform policies.” *Resource and Energy Economics* 32.4 (2010): 483-499.

Archer, D.W. and Shogren, J.F., 1996. Endogenous risk in weed control management. *Agricultural Economics*, *14*(2), pp.103-122.

Bockstael, Nancy E. “Modeling Economics and Ecology: The Importance of a Spatial Perspective.” *American Journal of Agricultural Economics* 78 (1996): 1168-1180.

Brock, William, and Anastasios Xepapadeas. 2008. “Diffusion-induced instability and pattern formation in infinite horizon recursive optimal control.” *Journal of Economic Dynamics and Control* 32 (9):2745-2787.

Brock, William, and Anastasios Xepapadeas. 2010. “Pattern formation, spatial externalities and regulation in coupled economic–ecological systems.” *Journal of Environmental Economics and Management* 59 (2):149-164.

Burnett, K., Kaiser, B., Pitafi, B.A. and Roumasset, J., 2006. Prevention, eradication, and containment of invasive species: illustrations from Hawaii. *Agricultural and Resource Economics Review*, *35*(1), p.63.

Costello, Christopher, Nicolas Quérou, and Agnes Tomini. "Private eradication of mobile public bads." *European Economic Review*94 (2017): 23-44.

Costello, Christopher, and Stephen Polasky. “Optimal harvesting of stochastic spatial resources.” *Journal of Environmental Economics and Management* 56.1 (2008): 1-18.

Costello, C., Springborn, M., McAusland, C., & Solow, A. (2007). Unintended biological invasions: Does risk vary by trading partner? *Journal of Environmental Economics and Management*, *54*(3), 262-276.

Eiswerth, Mark E., and Wayne S. Johnson. “Managing nonindigenous invasive species: insights from dynamic analysis.” *Environmental and resource economics* 23, no. 3 (2002): 319-342.

Epanchin-Niell, R.S. and Wilen, J.E., 2015. Individual and cooperative management of invasive species in human-mediated landscapes. *American Journal of Agricultural Economics*, *97*(1), pp.180-198.

Epanchin-Niell, Rebecca S., and James E. Wilen. “Optimal spatial control of biological invasions.” *Journal of Environmental Economics and Management* 63.2 (2012): 260-270.

Fenichel, Eli P., Timothy J. Richards, and David W. Shanafelt. “The Control of Invasive Species on Private Property with Neighbor-to-Neighbor Spillovers.” *Environmental and Resource Economics* (2014): 1-25.

Finnoff, David, Jason F. Shogren, Brian Leung, and David Lodge. “Take a risk: preferring prevention over control of biological invaders.” *Ecological Economics* 62, no. 2 (2007): 216-222.

Gopalakrishnan, S., McNamara, D., Smith, M.D. and Murray, A.B. "Decentralized management hinders coastal climate adaptation: the spatial-dynamics of beach nourishment." *Environmental and Resource Economics*, *67*(4) (2017), pp.761-787.

Grimsrud, K.M., Chermak, J.M., Hansen, J., Thacher, J.A. and Krause, K., 2008. A two-agent dynamic model with an invasive weed diffusion externality: An application to Yellow Starthistle (Centaurea solstitialis L.) in New Mexico. *Journal of environmental management*, *89*(4), pp.322-335.

Homans, Frances, and Tetsuya Horie. "Optimal detection strategies for an established invasive pest." *Ecological Economics* 70.6 (2011): 1129-1138.

Land, Ailsa H., and Alison G. Doig. "An automatic method of solving discrete programming problems." *Econometrica: Journal of the Econometric Society* (1960): 497-520.

Leung, Brian. David M. Lodge, David Finnoff, Jason F. Shogren, Mark A. Lewis, and Gary Lamberti. "An ounce of prevention or a pound of cure: bioeconomic risk analysis of invasive species." *Proceedings of the Royal Society of London B: Biological Sciences* 269.1508 (2002): 2407-2413.

Lockwood, Julie L. Martha F. Hoopes, and Michael P. Marchetti. *Invasion ecology.* John Wiley & Sons, 2013.

Mehta, Shefali V., Robert G. Haight, Frances R. Homans, Stephen Polasky, and Robert C. Venette. "Optimal detection and control strategies for invasive species management." *Ecological Economics* 61, no. 2 (2007): 237-245.

Olson, L.J. and Roy, S., 2002. The economics of controlling a stochastic biological invasion. *American journal of agricultural economics*, *84*(5), pp.1311-1316.

Pimentel, David, Rodolfo Zuniga, and Doug Morrison. "Update on the environmental and economic costs associated with alien-invasive species in the United States." *Ecological Economics* 52.3 (2005): 273-288.

Potapov, Alexei B., Mark A. Lewis, and David C. Finnoff. "Optimal control of biological invasions in lake networks." *Natural Resource Modeling* 20.3 (2007): 351-379.

Sanchirico, James N., et al. "Spatial management of invasive species: pathways and policy options." *Environmental and resource economics* 45.4 (2010): 517-535.

Sanchirico, James N., and James E. Wilen. "Bioeconomics of spatial exploitation in a patchy environment." *Journal of Environmental Economics and Management* 37, no. 2 (1999): 129-150.

Sanchirico, James N., and James E. Wilen. "Optimal spatial management of renewable resources: matching policy scope to ecosystem scale." *Journal of Environmental Economics and Management* 50, no. 1 (2005): 23-46.

Sharov, Alexei A. "Bioeconomics of managing the spread of exotic pest species with barrier zones." *Risk Analysis* 24.4 (2004): 879-892.

Smith, Martin D., James N. Sanchirico, and James E. Wilen. "The economics of spatial-dynamic processes: applications to renewable resources." *Journal of Environmental Economics and Management* 57.1 (2009): 104-121.

Figures

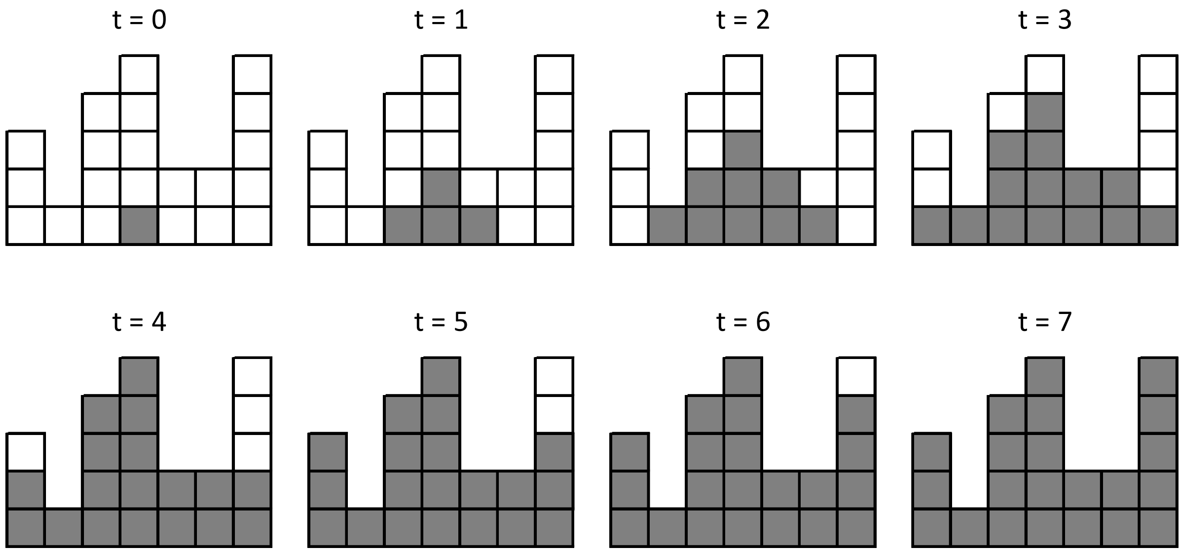


Figure 1 Spread dynamics with initial invasion intensity of one

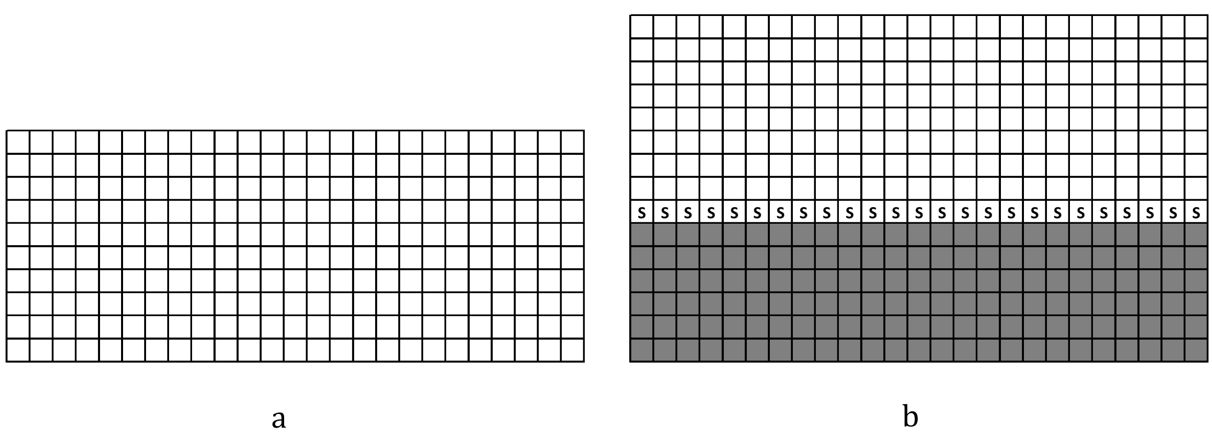


Figure 2: Two landscapes with different homogeneous carrying capacities and their resulting long run optimal control solutions. (Alpha = 1, 26 <= Gamma <=34)

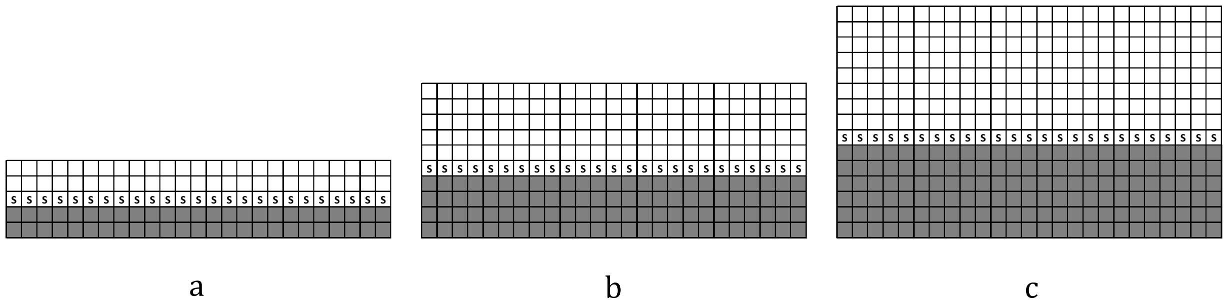


Figure 3: Three landscapes with different homogeneous carrying capacities and their resulting long run optimal control solutions. (Alpha = 1, Gamma >=57)

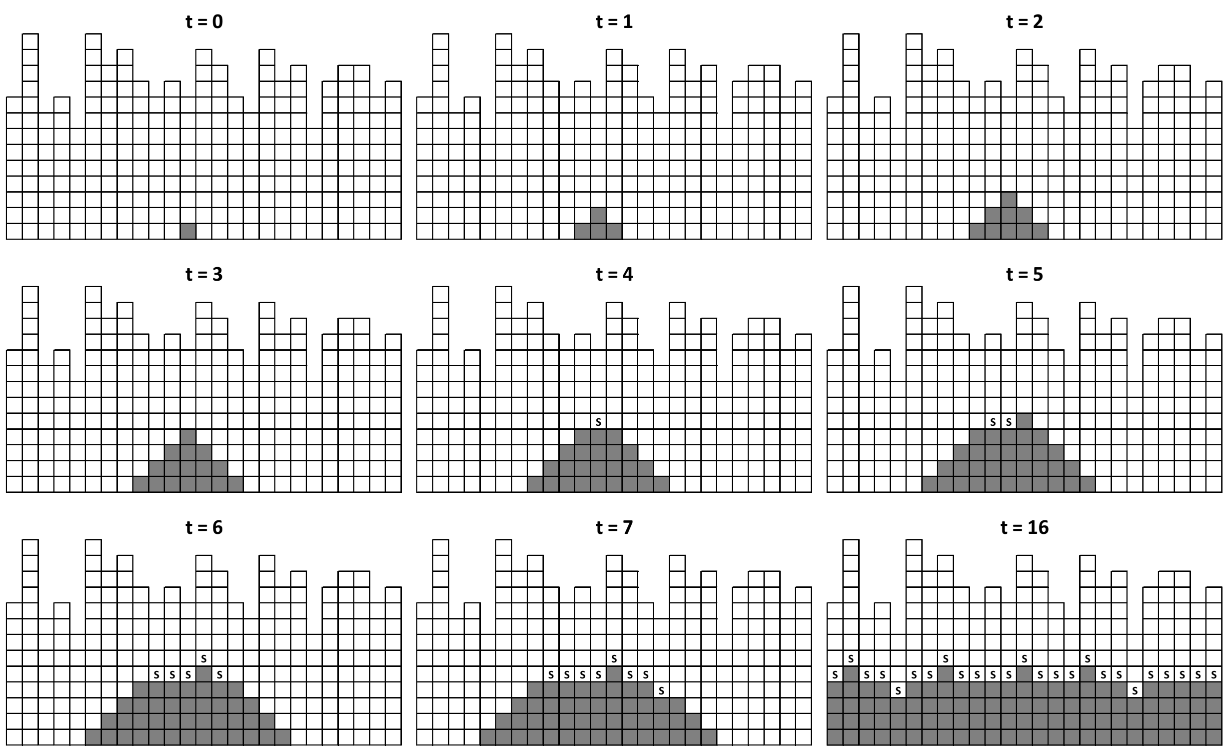
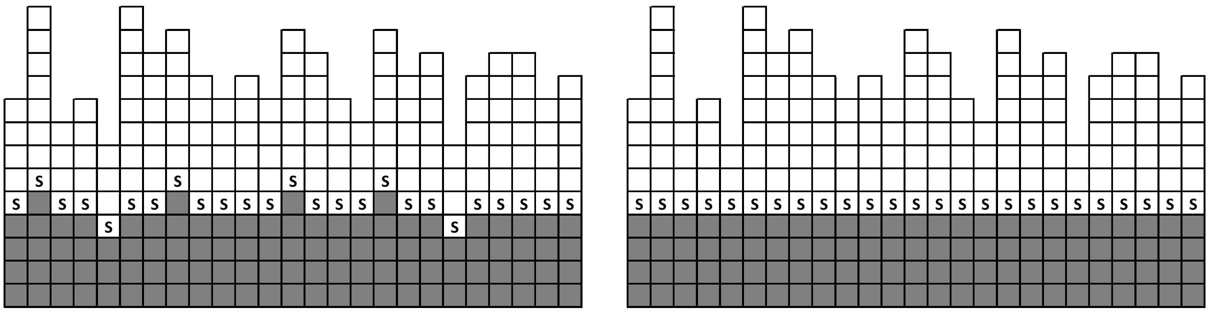
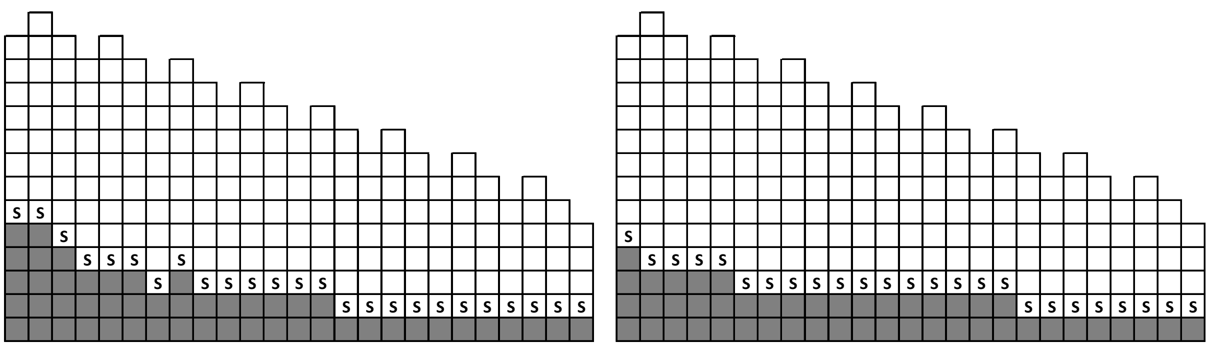


Figure 4: A landscape with heterogeneous carrying capacities and the pathway toward the long run optimal control solution. (Alpha = 1, Gamma >= 34)



a b

Figure 5: A heterogeneous landscape with a. the optimal control solution when heterogeneity is recognized, and b. the sub-optimal solution when homogeneity is assumed.
(Alpha = 1, Gamma >= 35)



a b

Figure 6: A heterogeneous landscape with spatially varying baseline patch values. In panel a. heterogeneous carrying capacities are recognized, in panel b. homogeneity is assumed. For both panels, the spatially varying baseline patch values are recognized.
(Alpha = 1, Gamma >=52, $P_i = \{1{:}\,0.5{:}\,13\}$)

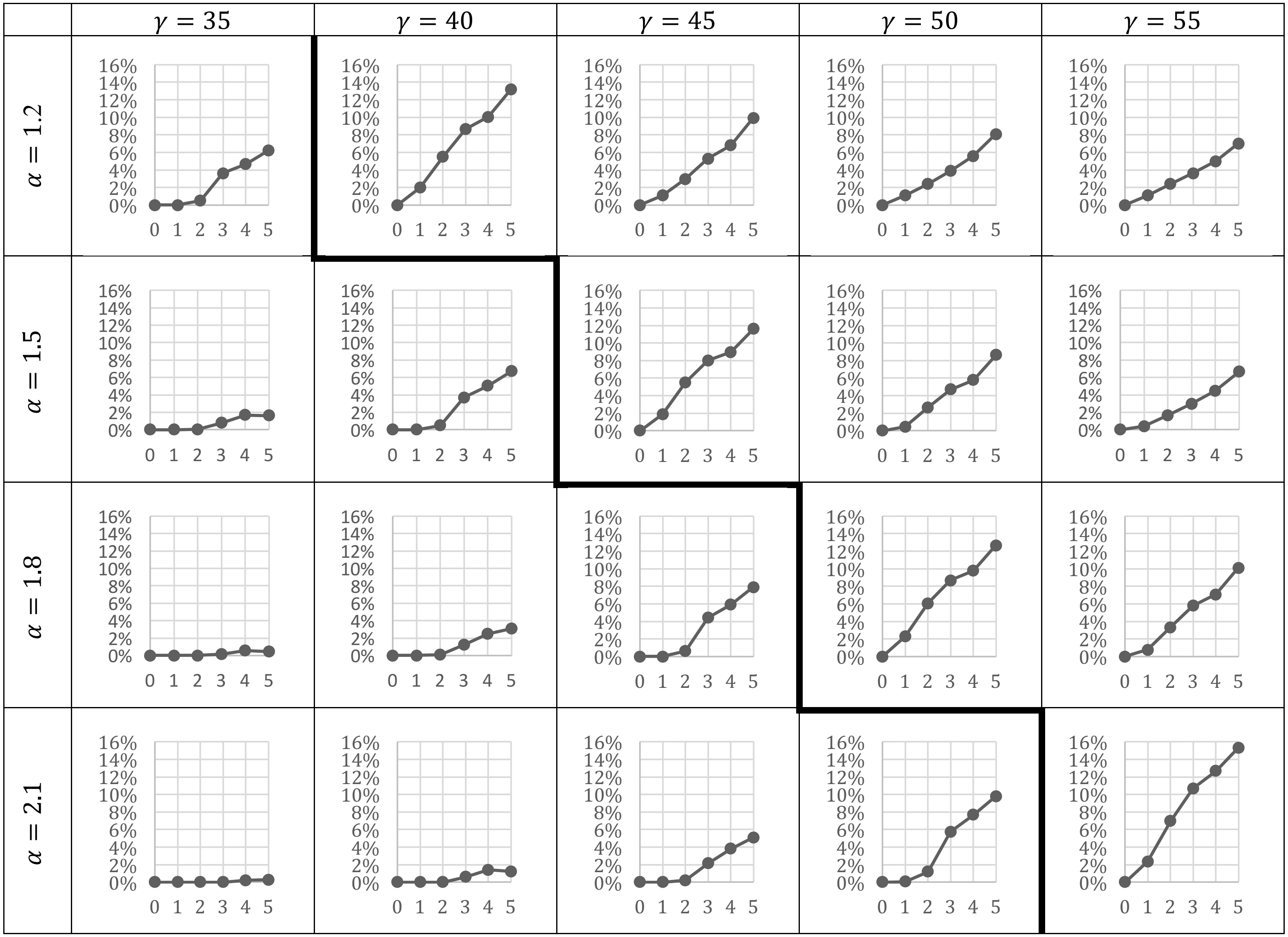


Figure 7: Inefficiencies of homogeneous assumption in landscapes with increasingly heterogeneous carrying capacities

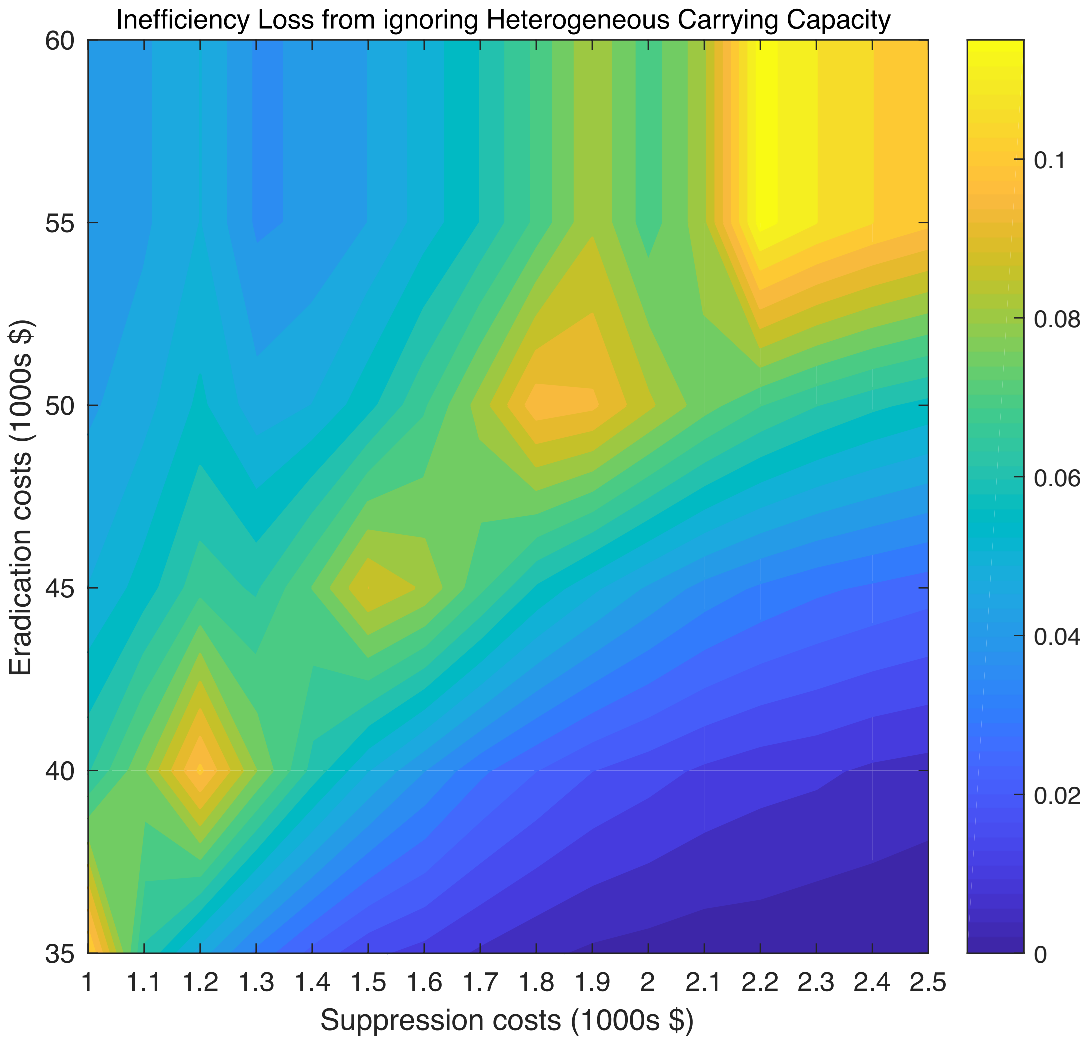
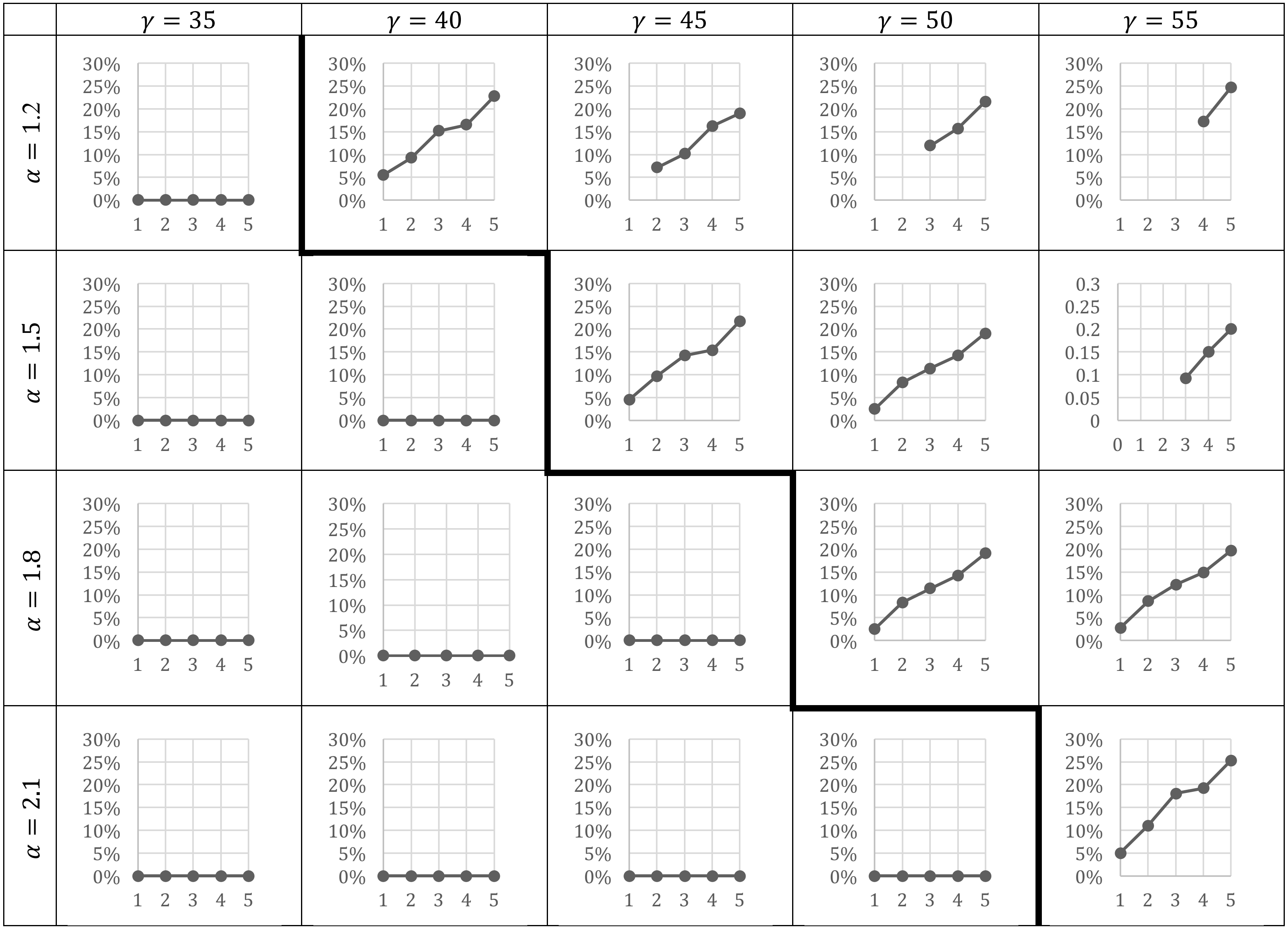


Figure 8: Inefficiency Costs from Ignoring Spatial Heterogeneity in Carrying Capacities



$\gamma = 35$
$\gamma = 40$
$\gamma = 45$
$\gamma = 50$
$\gamma = 55$
$\alpha = 1.2$
$\alpha = 1.5$
$\alpha = 1.8$
$\alpha = 2.1$

Figure 9: Average inefficiencies when eradication is the optimal strategy

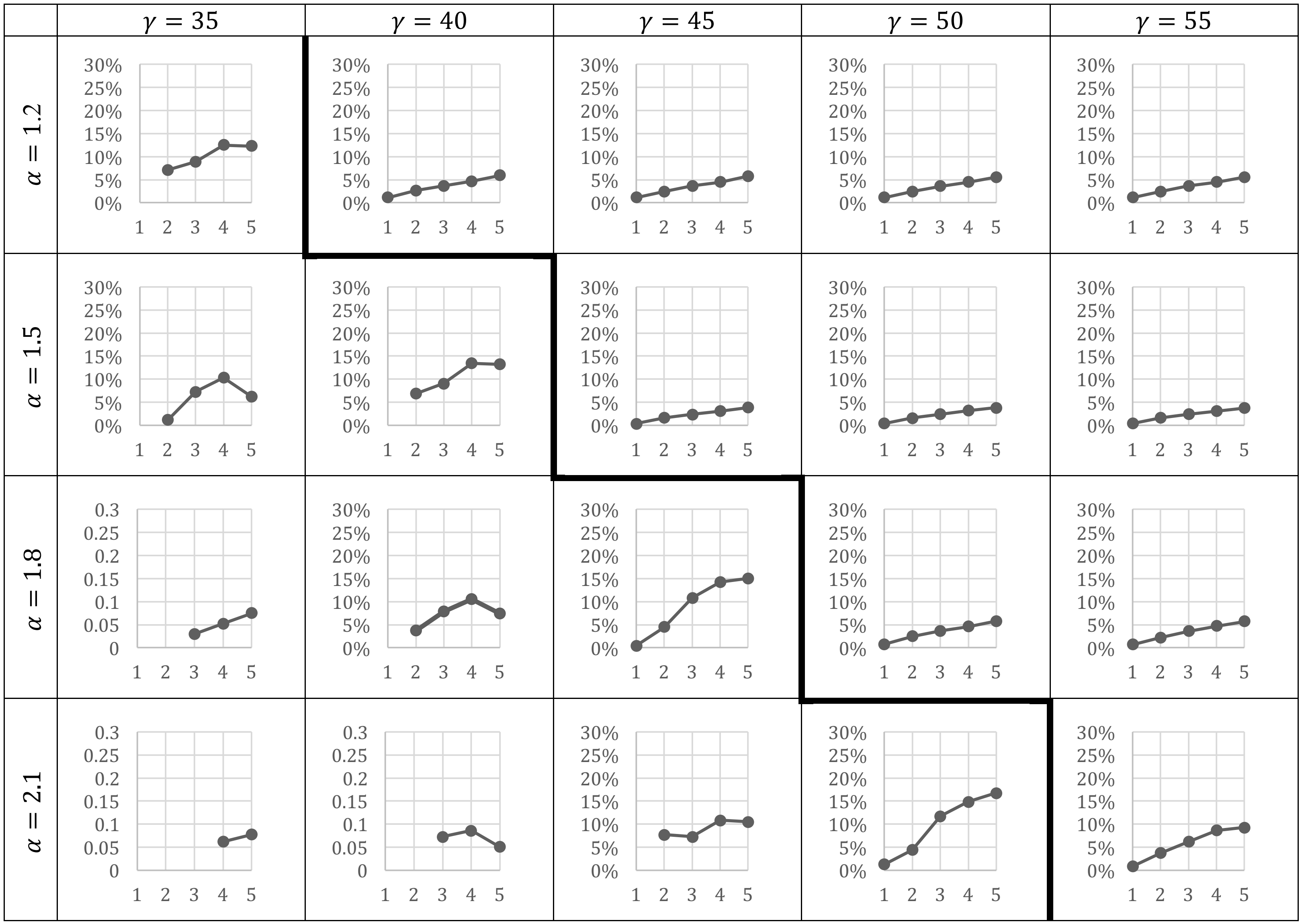


Figure 10: Average inefficiencies when suppression is the optimal strategy.

Appendix A: Continued exploration of heterogeneous landscapes

A.1 Changes in the functional form of suppression cost

Appendix A.3 shows that, in a homogeneous landscape, when the suppression cost function is assumed to be linear with respect to carrying capacity and is not related to the invasion intensity, the optimal suppression strategy is akin to a “bang-bang” solution, where the invasion is either suppressed to a level of one or not suppressed at all. This is not necessarily the case when the carrying capacities are allowed to vary over space.

Figure B.1.2 shows that five distinct optimal control solutions exist for the heterogeneous landscape shown in figure B.1.1. These solutions are shown in figure B.1.3. While the suppress to one and the no management strategies are still optimal under certain parameterizations, two new suppression based optimal control solutions now exist in between them.

Interior suppression solutions under linear suppression costs are not limited to slight variations in intensity. In a more stylized landscape shown in figure B.1.4, we see another interior suppression solution being optimal with intensity levels allowed to increase as high five in the high carrying capacity center of the landscape.

Clearly, heterogeneous carrying capacities affect the optimal control strategies even when suppression costs are linear with respect to carrying capacity.

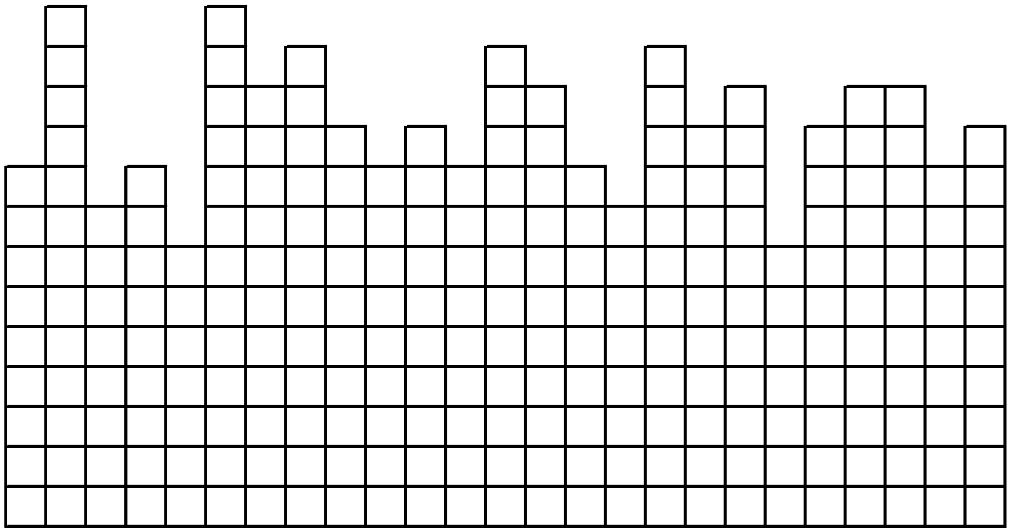


Figure A.1.1: Heterogeneous Landscape

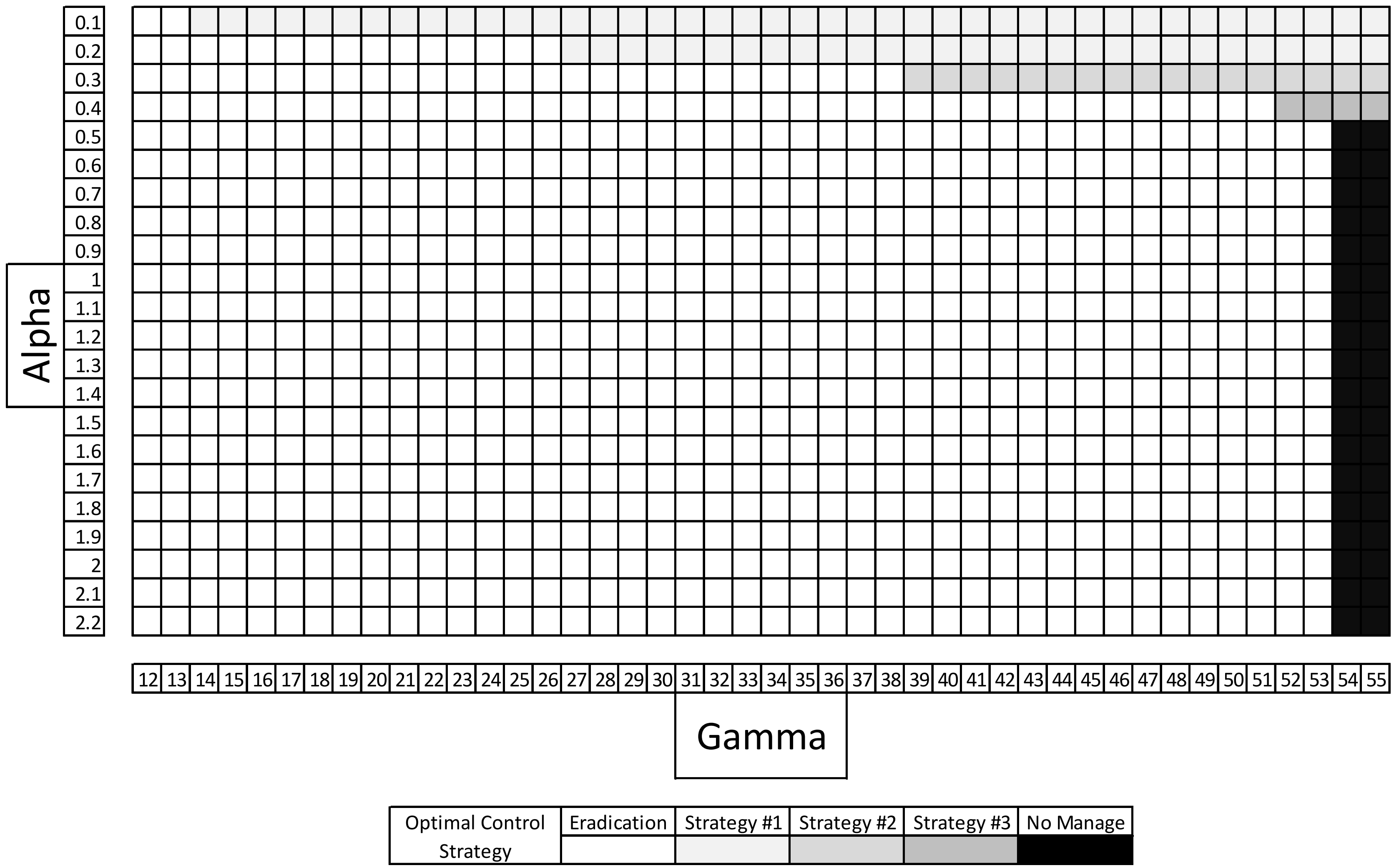
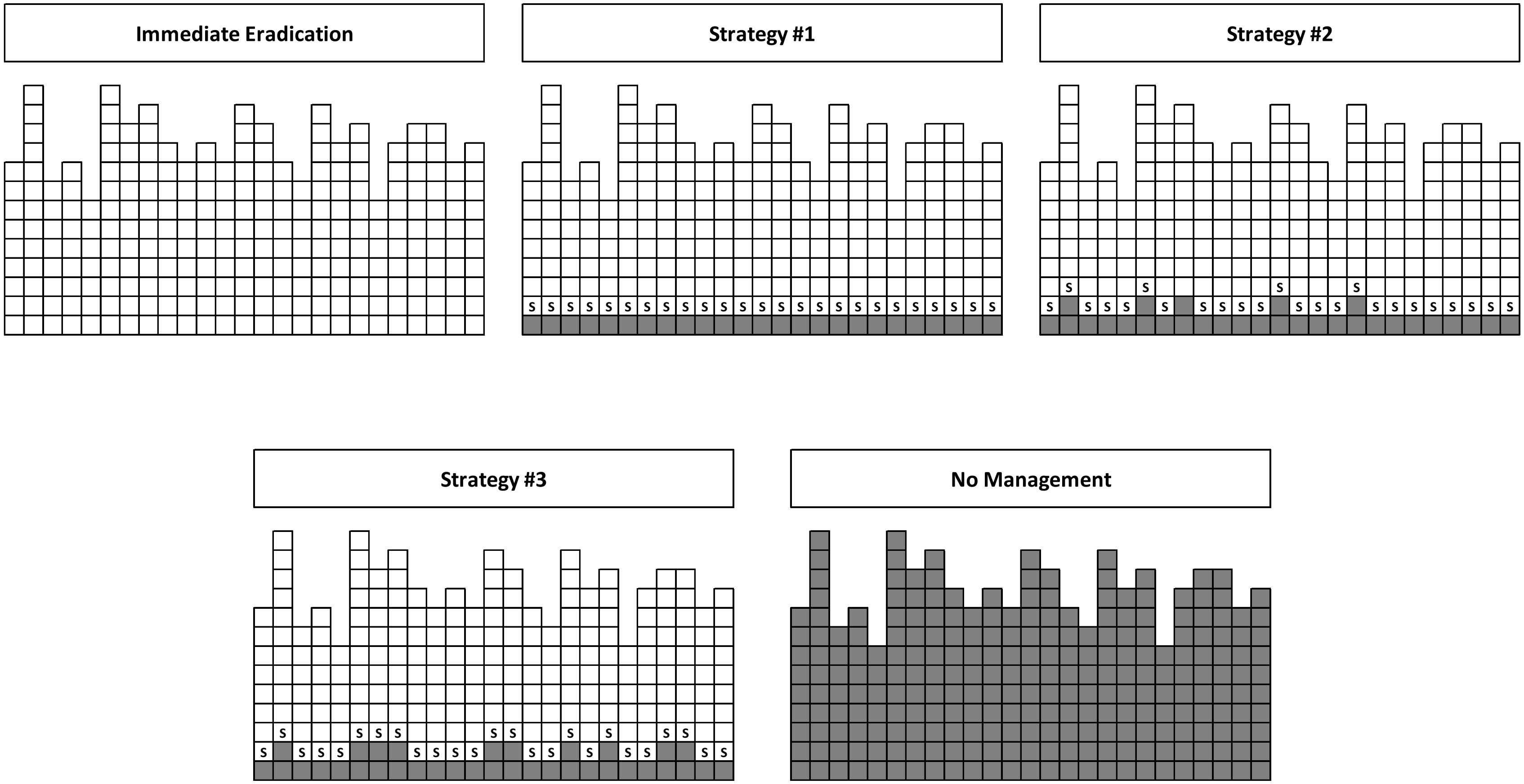


Figure A.1.2: Solutions Under Variety of Parameterizations when Suppression Cost is Linear



gure A.1.3: Optimal Control Strategies from Figure A.4.2.

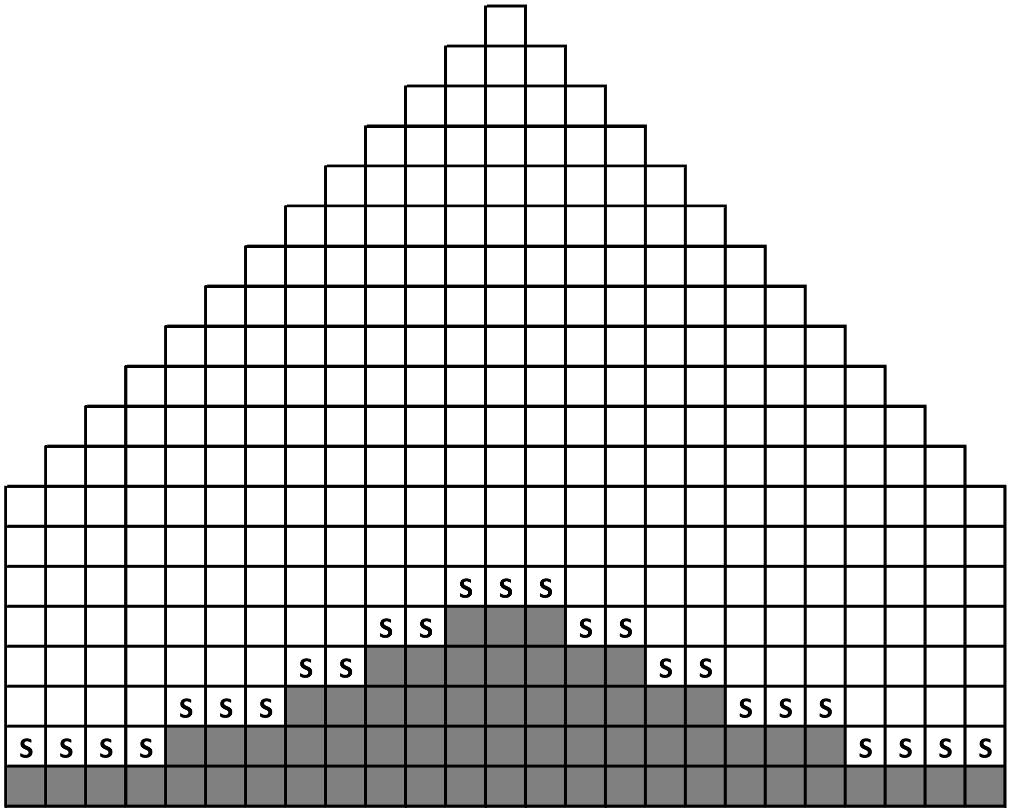


Figure A.1.4: Optimal Control Steady State for a More Stylized Heterogeneous Landscape and Linear Suppression Costs
$(\alpha = .3, \gamma \geq 30)$

A.2 Optimal Management with Density Dependent Growth

Incorporating a density-dependent population growth for invasive populations into a binary integer problem is computationally challenging and limits the dimensionality of the problem. Constraints are no longer as simple as those for linear spread and growth because the intensity of invasion within each patch now relies upon the total intensity of the invasion in neighboring patches. In an integer programming problem, to determine when a given cell is invaded, we need to specify every possible pattern of invasion that results in the invasion of the cell. Because of this complexity, we need to limit the spatial extent of the landscape and the time horizon of the problem. We simulate a landscape with 5 ecological patches and a maximum carrying capacity of 8 units with diffusion to test for the robustness of the model described in Section 3.2. We solve the problem in equation (1) with the following transition dynamics:

$$x_{i(t+1)} = \min\left\{x_{it} + \frac{(x_{(i-1)t} - x_{it}) + (x_{(i+1)t} - x_{it})}{2}, k_i\right\} \tag{5}$$

Under this growth function, the intensity of the invasive stock in any given patch depends not only upon the stock in the previous time period, but also upon the relative density in neighboring patches. The growth resulting from this function is shown in figure B1. Notice that growth is more rapid under this specification.

When the previous landscape of 5 homogeneous patches with carrying capacity 8 is simulated over a 13 period time horizon with $\alpha = 1$, the optimal management pathway is shown in figure B2. Just like the linear model, the optimal control strategy in this homogeneous landscape is to maintain a uniform level of intensity across space. Similarly, the optimal control path in a landscape with heterogeneous carrying capacities drawn from normal distribution centered at 8 (shown in figure B3) is heterogeneous and is comparable to the pattern in Figure 3.

Figure A.2.1: Density Dependent Growth Process in Homogeneous Landscape

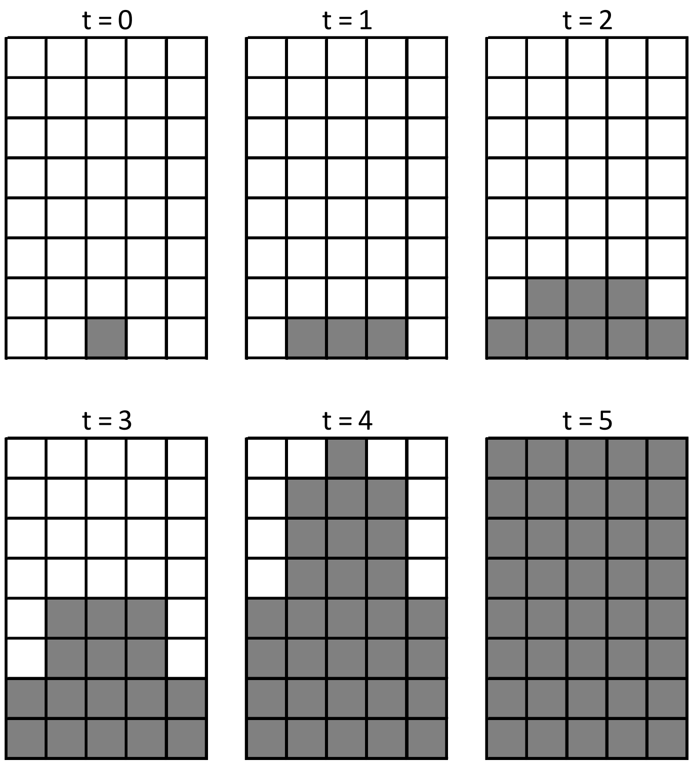


Figure A.2.2: Density Dependent Growth Process

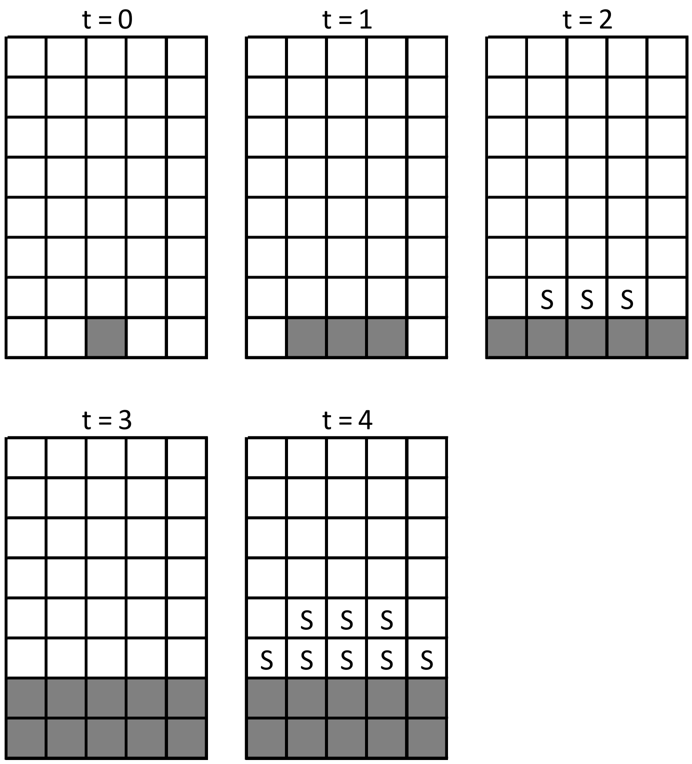


Figure A.2.3: Density Dependent Growth Process in Heterogeneous Landscape