Learning, Growth and Climate Feedbacks

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Broad Question

- How do possible future climate disasters affect near term optimal climate policy (optimal emissions)?

Climate Policy and the Discount Rate

- The ongoing worldwide climate policy debates: shall we
  - implement stringent control over GHG emissions now, or
  - wait until we know more about climate change?
- Since damages due to climate change would most likely happen in the far future, the answer depends crucially on rate of return r used to discount future utilities:
  - $r \downarrow \Rightarrow$ more stringent GHG control now (the Stern Review)
  - $r \uparrow \Rightarrow$ wait and see (Nordhaus DICE model)

Discount Rate and Economic Growth

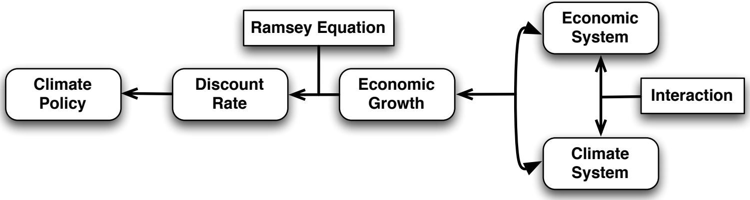
Suppose the economic growth rate $g \sim N(\mu_g, s_g^2)$

$$r = \rho + \sigma\mu_g - \frac{1}{2}\sigma^2 s_g^2 \tag{1}$$

- According to the Ramsey equation, three factors determine the rate used to discount consumption:
  - the rate of pure time preference ρ
  - relative risk aversion σ
  - the stochastic economic growth rate g

Economic Growth and Climate

- future climate damages $\uparrow \Rightarrow$ future $\mu_g \downarrow \Rightarrow r \downarrow$
  $\Rightarrow$ optimal emissions now$\downarrow$
- Want to examine how and whether future climate disasters would affect economic growth, hence near term climate policy.



Uncertainty

"Fear comes from uncertainty. When we are absolutely certain, whether of our worth or worthlessness, we are almost impervious to fear."

– William Congreve

- Assuming an exogenous growth rate, Weitzman argues
  - Fat tails reflect the "deep structural uncertainty for the low-probability, high-impact catastrophes".
  - fat tails in climate system $\Rightarrow s_g \uparrow \Rightarrow r \downarrow \Rightarrow$ act now.
  - Bayesian learning would not help to thin down the fat tails ($s_g \downarrow$).

Specific Questions

- Would uncertainties in climate change necessarily lead to fat tails in growth rate?
- How would learning help to resolve these uncertainties?
  - *Weitzman (2009) :"... It is inherently difficult to learn from finite samples alone enough about the probabilities of extreme events to thin down the bad tail of the PDF because, by definition, we don't get many data point observations of such catastrophes. "*

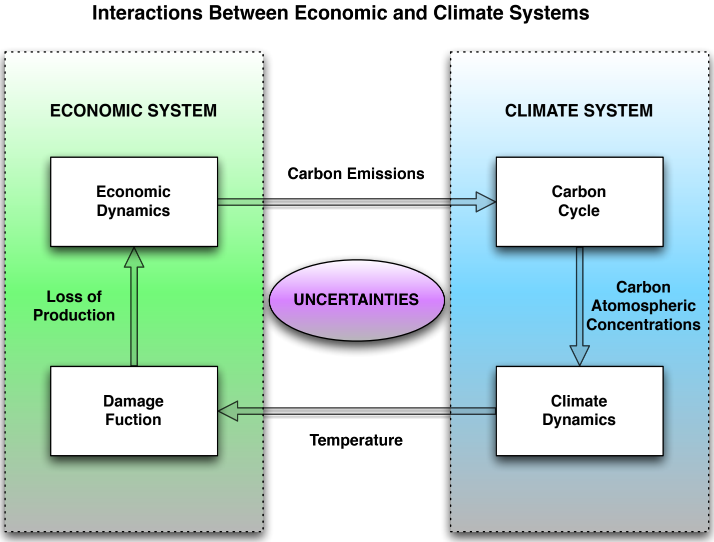
Objectives

- Build an integrated assessment model (IAM) to capture the need to use a limited amount of information to update the distribution of a complex system in order to set climate policy.
- The model features:
  - uncertainty about positive climate feedbacks
  - Bayesian learning about the climate feedback parameter (fat tails)
  - GHG emissions as tied to production
  - a balanced growth path with exogenous growth in labor productivity and population

Objectives (cont.)

- Calibrate the model to real economic and climate data.
- Examine the dynamics of resolution of uncertainty.
  - Do fat tails matter?
  - Can we learn fast enough to avert a disaster? If so, take little action now.
- Use the model to provide quantitative evaluations of the near term climate policies.

Model Outline



The Economic System

- The production technology is such that:

$$Q_t = F(K_t, A_t L_t) = K_t^{\psi}(AL_t)^{1-\psi} \tag{2}$$

- Unabated pollution is an exogenous proportion $1/B_t$ of output

$$E_t = (1 - u_t)\frac{Q_t}{B_t} \tag{3}$$

The Economic System

- We assume a convex cost function:

$$C(u_t) = 1 - (1 - u_t)^{\epsilon} \tag{4}$$

- Y_t is the output net of abatement costs

$$Y_t = (1 - C(u_t))Q_t \tag{5}$$

The Climate System

- GHG accumulates according to:

$$M_{t+1} - MB = (1 - \delta_m)(M_t - MB) + \gamma E \tag{6}$$

- which in turn changes the temperature

$$T_t = T_{t-1} + \frac{1}{\alpha}\left(F_t - \frac{T_{t-1} - \Gamma}{\lambda}\right) + \nu_t \tag{7}$$

The Climate System

where

- T_t is the annual global temperature ($^\circ C$) deviation from 1961-90 average at time t
- Γ is the preindustrial temperature in deviations (known)
- Ω is the radiative forcing parameter (known)
- α is the heat capacity of ocean (known)
- λ is climate sensitivity, which is a measure of how responsive the temperature of the climate system is to a change in the radiative forcing (unknown)

Facts about the Temperature Model

(1) If GHG concentration stays at preindustrial level, temperature converges to preindustrial temperature. That is, in steady state, with $\bar{M} = MB$,

$$\bar{T} = \Gamma \tag{8}$$

(2) The steady state change in global mean near-surface air temperature that would result from a sustained doubling of the atmospheric (equivalent) CO2 concentration, $\bar{M} = 2MB$, is

$$\triangle T_{2\times} = \lambda\Omega = 4.39\lambda \tag{9}$$

The Feedback Parameter

The temperature model can be reorganized as an AR(1) process:

$$T' = \beta_1 T + \beta_2 \log(m') + \nu' \tag{10}$$

here

- $T = T - \Gamma$ is the temperature deviation from preindustrial level
- $\beta_2 = \frac{\Omega}{\alpha \log(2)}$
- β_1 is the unknown feedback parameter: $\beta_1 \sim N(\mu, \xi)$
- ν is the stochastic shock: $\nu' \sim N(\mu_\nu, \sigma_\nu^2)$

Damage Function

Damages to total output are due to temperature. We borrow the damage function from Weitzman (2009):

$$D(T) = \exp\left(-b_1 T^{b_2}\right) \tag{11}$$

The Recursive Problem

The social planner tries to maximize the net present value of population utility:

$$v(s) = \max_{k',E} \left\{ \frac{(c)^{1-\sigma}}{1-\sigma} + \hat{\beta} E\left[v(s')\right] \right\} \tag{12}$$

where $s = [k, m, T, \mu, \xi]$

subject to

- $c = D(T)f(k, E) + (1 - \delta_k)k - (1 + \eta)(1 + \phi)k'$
- $m' = 1 + (1 - \delta_m)(m - 1) + \left(\frac{\gamma}{MB}\right) E$
- $T' = \beta_1 T + \beta_2 \log(m') + v'$

Bayesian Learning

Bayesian rule gives two additional law of motions for (μ, ξ). Posterior distribution of β_1 is still normal with:

$$\mu' = \frac{\mu + \xi p_\nu TH'}{1 + \xi p_v T^2} \quad (13)$$

$$\xi' = \frac{\xi}{1 + p_v \xi T^2} \quad (14)$$

where

$$H' = \beta_1 T + v' \quad (15)$$

Learning is endogenous. Each period new climate records arrive and the social planner update the prior on the feedback parameter β_1.

From Feedbacks to Climate Sensitivity

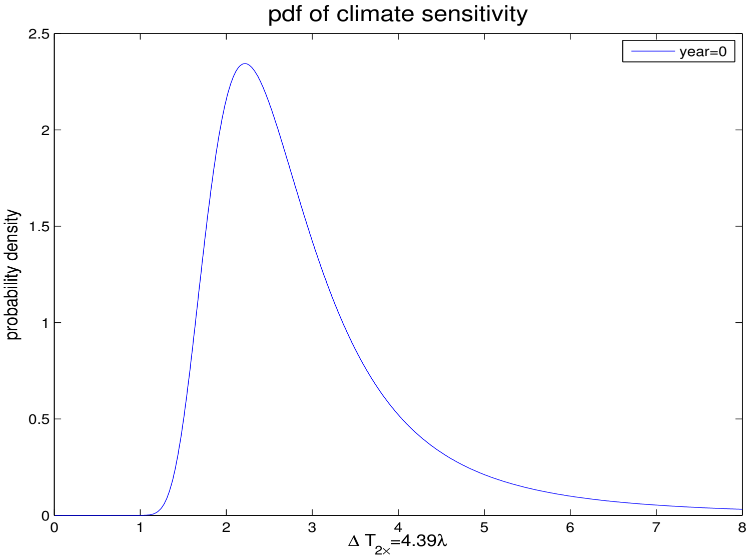
In the temperature model, we have

$$\beta_1 = 1 - \frac{1}{\alpha\lambda} \sim N(\mu, \xi) \tag{16}$$

We can then compute the pdf for climate sensitivity from Jacobian transformation

$$h_\lambda(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} \frac{\frac{1}{\alpha}}{\lambda^2} exp\left[-\frac{1}{2}\left(\frac{1 - \frac{1}{\alpha\lambda} - \mu}{\sigma}\right)^2\right] \tag{17}$$

Visualization of the Prior



Solution Algorithm

We use function iteration and Chebychev approximation to solve the problem.

$$\hat{v}_i(s;\chi_{i-1}) = \max_c u(s,c) + \hat{\beta} E_{\beta_1,\nu}\left[\Phi[g(s,c,\beta_1,\nu);\chi_{i-1}]\right] \quad (18)$$

$$\chi_{i-1} = \arg\min_{\chi} \|\Phi(s;\chi) - \hat{v}_i(s,\chi_{i-1})\| \quad (19)$$

The following summarizes the algorithm for finding the value function.

- 1. Compute χ_0, the parameter vector which minimizes the distance between v_0 and Φ.
- 2. Compute $\hat{v}_1$ by computing the optimal controls for the approximate value function Φ.
- 3. Compute χ_1, the parameter vector which minimizes the distance between $\hat{v}_1$ and Φ.
- 4. Repeat step (2) until $\|\hat{v}_i - \hat{v}_{i-1}\| < \eta^*$.

Simulations

Table: Two Learning Experiments

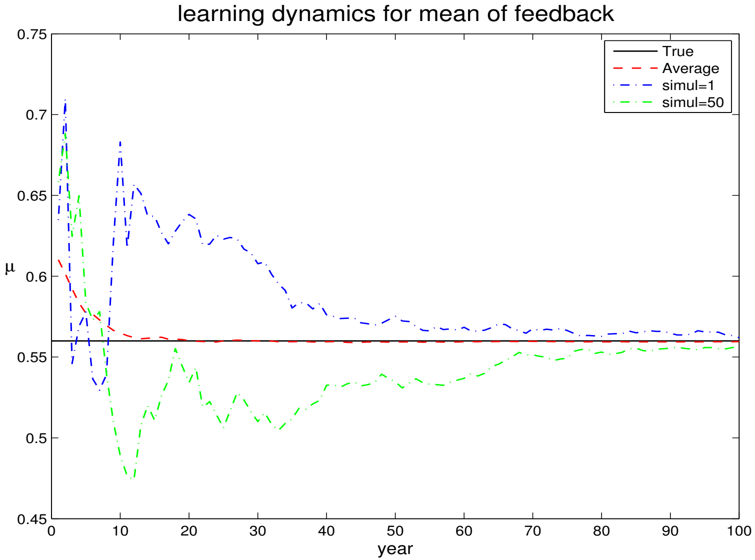
	Experiment 1	Experiment 2
ture Climate Sensitivity $\Delta T^*_{2\times}$	2.2289	4
true Feedback β_1^*	0.56	0.7481
piror for β_1	(0.62,0.0169)†	(0.62,0.0169)
number of simulations	500	500
number of years simulated	500	500

† from Colman(2008), which calculated (μ,ξ) from a suite of GCM simulations.

Observations

- It takes a long time to learn where the true mean is.
  - When $\Delta T^*_{2\times} = 2.23$, average time to resolve uncertainty ($\Pr(2.13 \leqq \Delta T_{2\times} \leqq 2.33) \geqq 95\%$) is 76 years.
  - When $\Delta T^*_{2\times} = 4$, average time to resolve uncertainty ($\Pr(3.9 \leqq \Delta T_{2\times} \leqq 4.1) \geqq 95\%$) is 132 years.
- Fat tails are diminished quickly!
  - When $\Delta T^*_{2\times} = 2.2298$, average time to rule out extreme climate sensitivity ($\Pr(\Delta T_{2\times} > 4) \leqq 1\%$) is 4 years.
  - When $\Delta T^*_{2\times} = 4$, average time to rule out extreme climate sensitivity ($\Pr(\Delta T_{2\times} > 8) \leqq 1\%$) is 7 years.

Experiment 1



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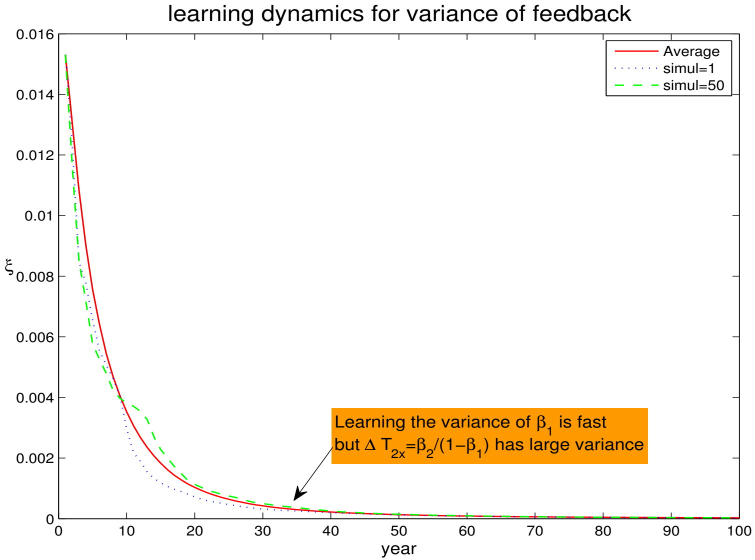

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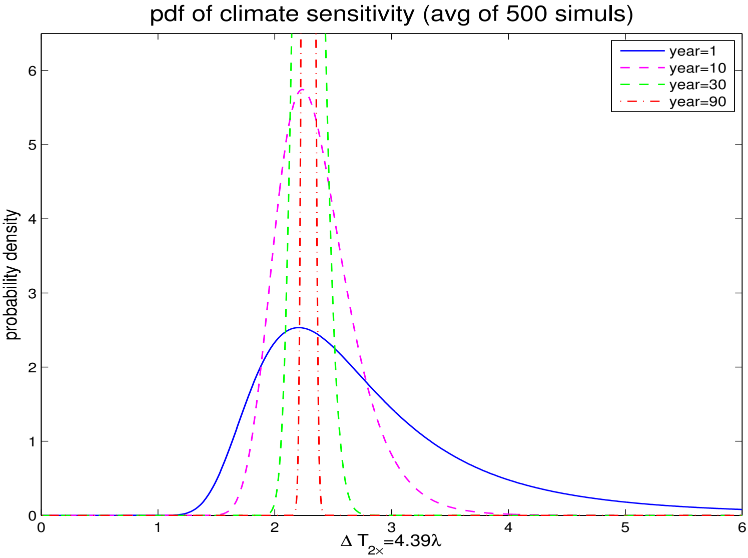

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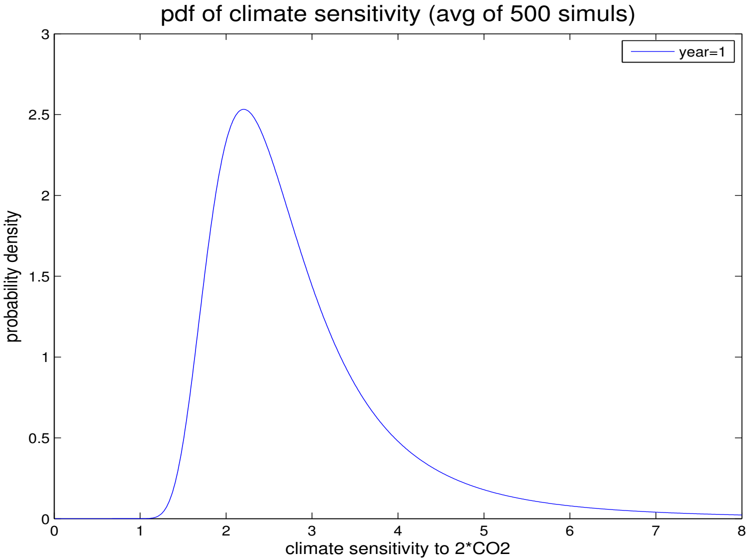

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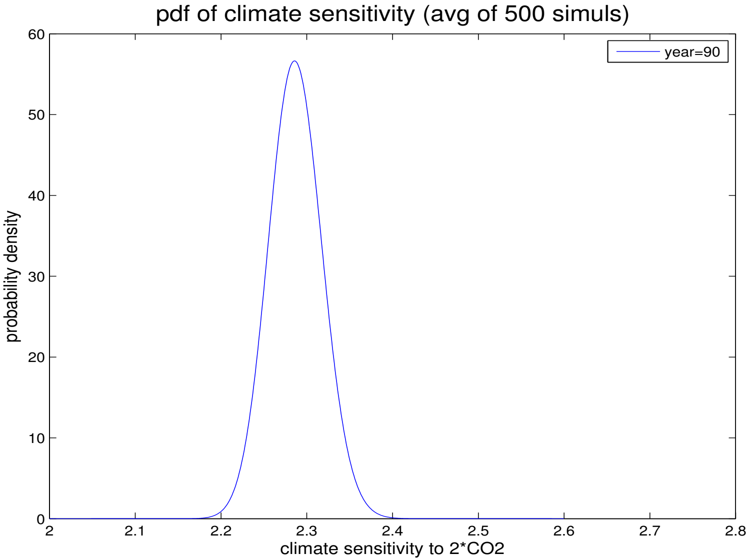
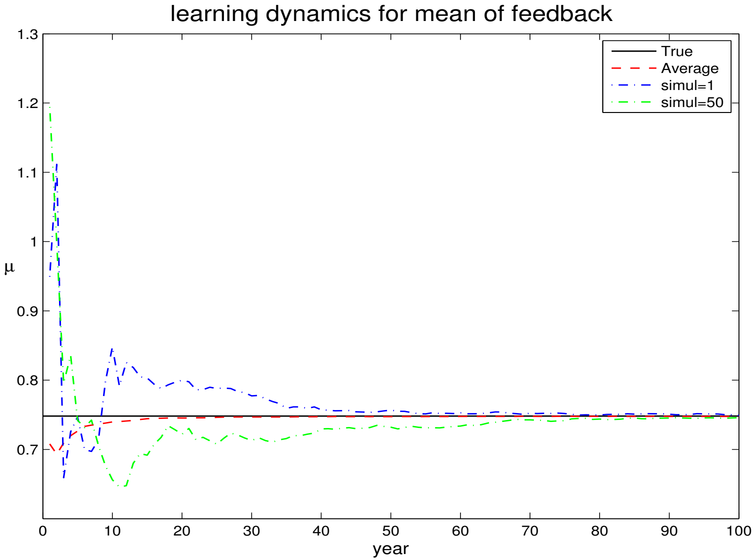


Experiment 2



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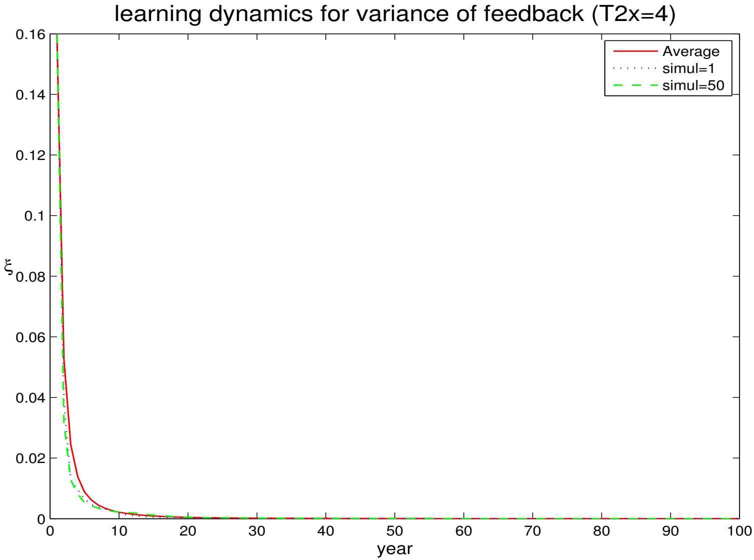

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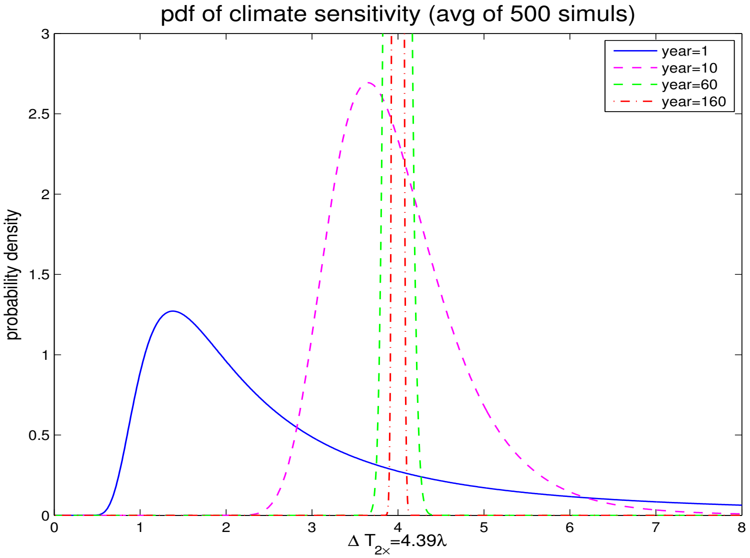

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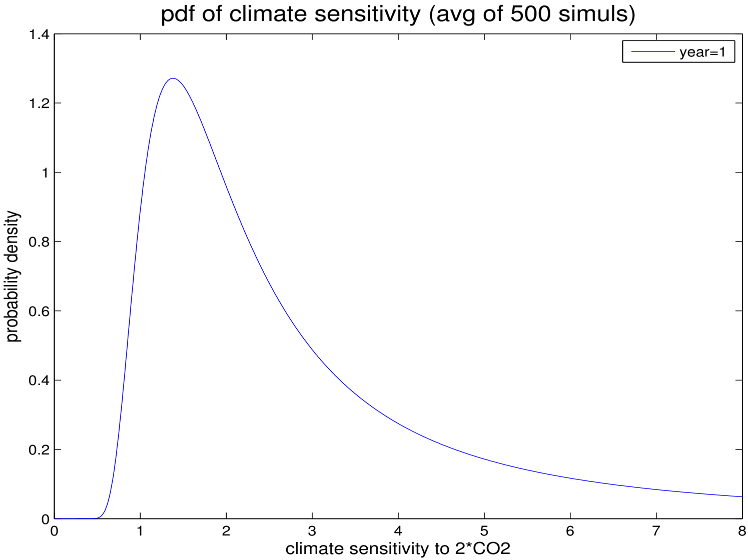

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