Contract Duration and Extraction with Hydraulic Fracturing

S.M. Asif Ehsan
Gregory S. Amacher
Klaus Moeltner

Virginia Tech.

Outline of Topics

Motivation for Paper

- Hydraulic fracturing is increasingly being used to extract natural gas and oil.

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- Concerns regarding harmful impacts on environment and health have been increasing.
- Public may be exposed to damages due to gas migration, contaminant transport and waste water discharge.
- Despite the heterogeneity in extractors and difference in technology, same five-year initial contract length is used in hydraulic fracturing.
- It is likely that this contract length will not reflect all off site costs.

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- We then treat contract length as endogenously determined.
- We explore how the difference in regulator's preferences for hydraulic fracturing warrants different tax rate on extraction.

Literature Review

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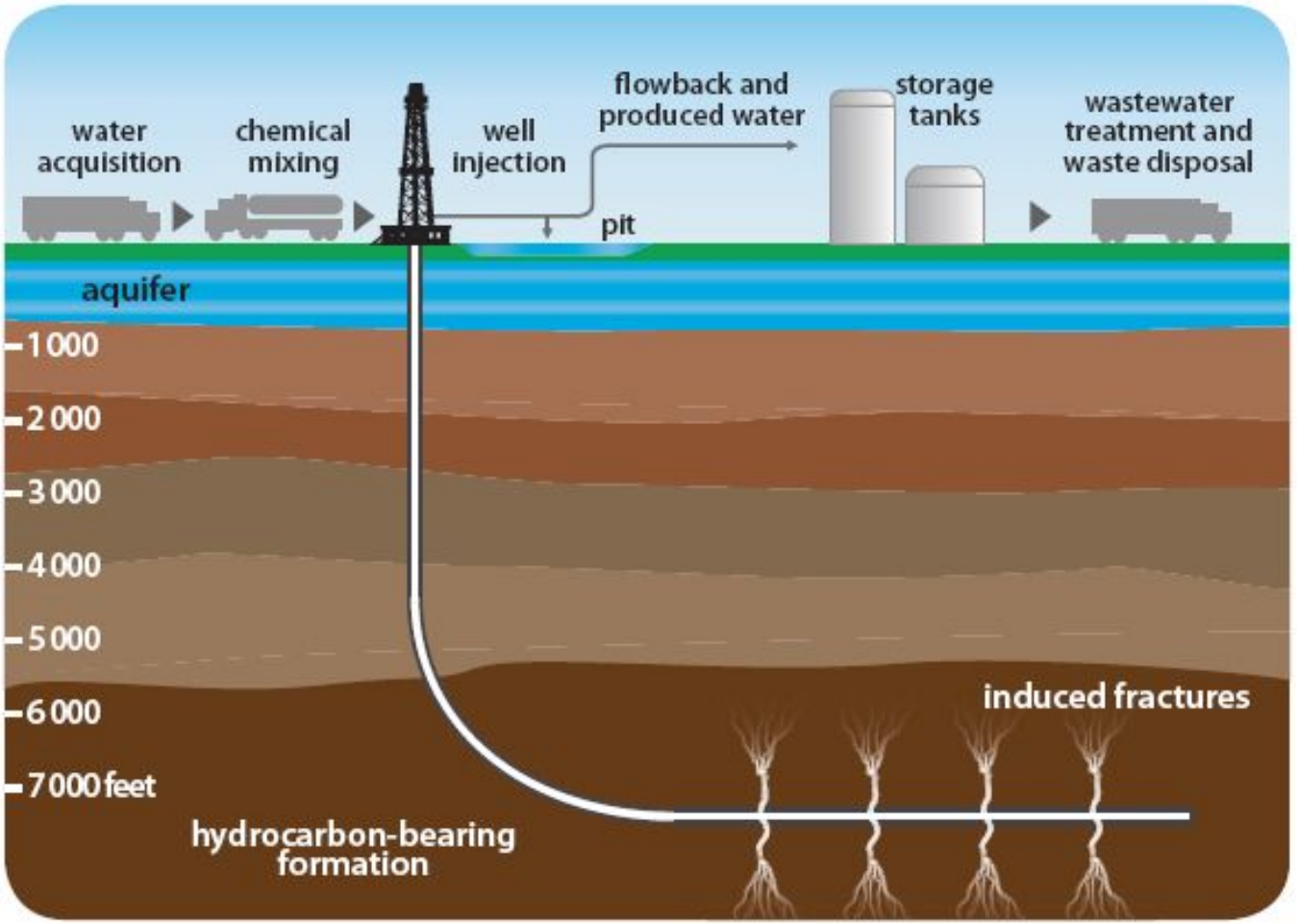
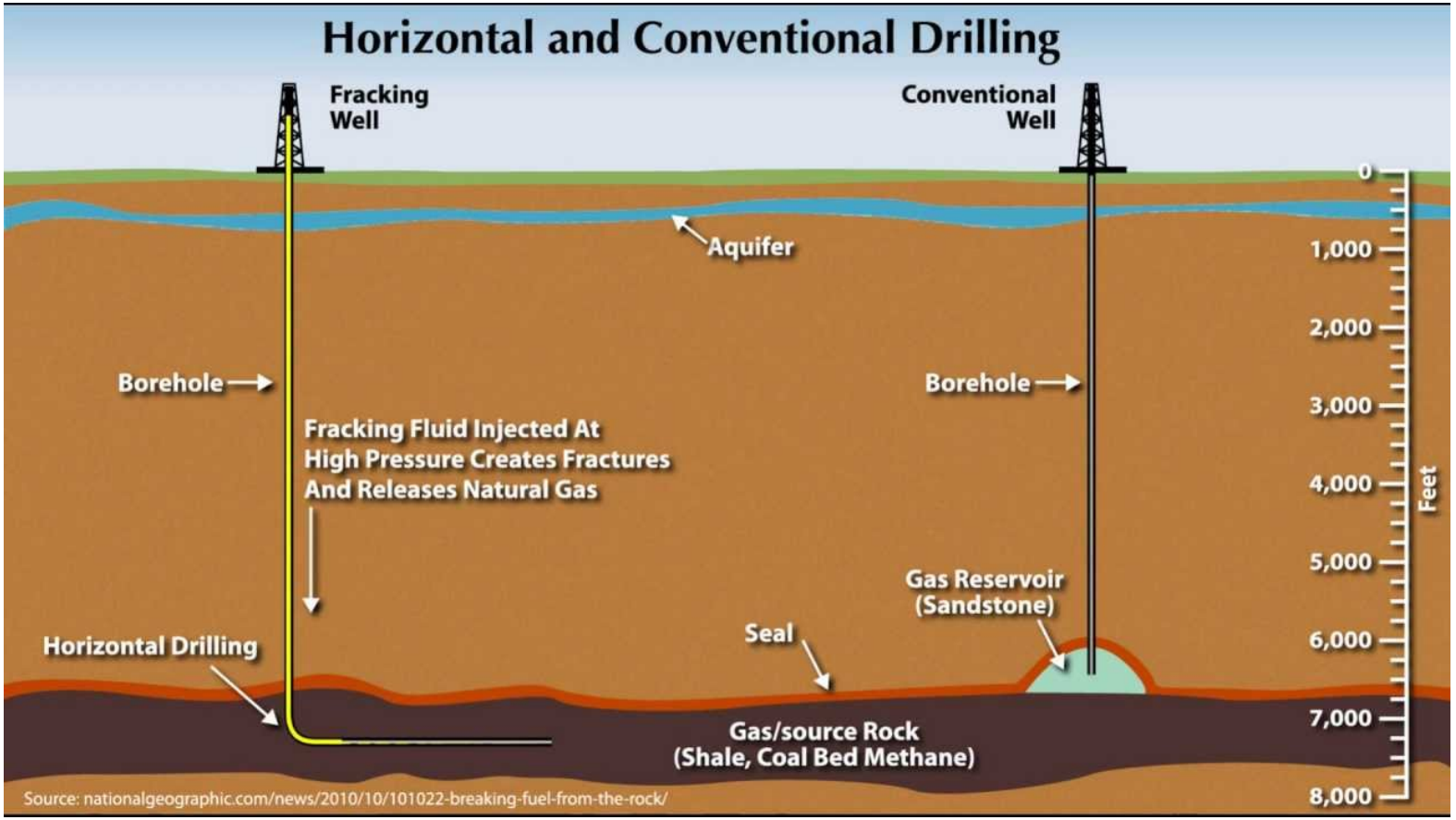
- Regulations to reduce environmental harm are inadequate for hydraulic fracturing (Holahan and Arnold 2013).
- A shorter period of commitment increases extraction rate Reinganum and Stokey (1985).
- Uncertainty and a smaller initial stock lead to a more conservative extraction over time Kumar (2002).

Process of Hydraulic Fracturing



Horizontal vs. Conventional Drilling



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- First we assume the contract length is given.
- Then we treat contract duration as endogenous.
- Explore the extraction paths $q(t)$ and contract length T.

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- The cost of extraction $c[q(t)]$ is convex and increasing in extraction rate $q(t)$.

Exogenous Contract Duration

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- The SP's problem under this scenario is:

$$\max_{q(t)} \int_0^T e^{-vt} \{pq(t) - c[q(t)] - D[q(t), l] - E[q(t)]\} dt + \varphi(x_T, T) e^{-vT}$$

$$s.t. \quad \dot{x}(t) = -q(t), \;\; x(0) = x_0, \;\; x(T) = x_T$$
$$0 \leq q(t) \leq x(t), \;\; x(t) \geq 0$$
$$\int_0^T q(t)dt = x_0 - x_T$$

Exogenous Contract Duration

Private Extraction Firm:

- The CLO solves the following problem for the optimal l:

$$\max_{l \in [0,L]} \int_0^T e^{-vt} \left\{ r(l) + R\left[q(t)\right] - D\left[q(t), l\right] \right\} dt$$

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- The private extractor's problem becomes:

$$\max_{q(t)} \int_0^T e^{-vt}\left\{pq(t)-c\left[q(t)\right]-r(I)-R\left[q(t)\right]\right\}dt$$
$$S.T.\ \ \dot{x}(t)=-q(t),\ \ x\left(0\right)=x_0,\ \ x\left(T\right)=x_T$$
$$0\leq q(t)\leq x(t),\ \ x(t)\geq 0 \qquad \forall t$$
$$\textstyle\int_0^T q(t)dt=x_0-x_T$$

Exogenous Contract Duration

Optimal paths for $q(t)$ and $x(t)$ are dictated by the following equations:

- For the SP problem:

$$\frac{\left[-c''\left[q(t)\right] - D_{qq}\left[q(t), I\right] - E_{qq}\left[q(t)\right]\right]\dot{q}(t)}{p - c'\left[q(t)\right] - D_q\left[q(t), I\right] - E_q\left[q(t)\right]} = v \tag{1}$$

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- For the private extraction firm problem:

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- If the second order damage and externality are small and royalty payments are constant, the SP chooses a lower extraction path than the private extractor.
- For both the SP and the private extractor model price, initial stock and contract duration have positive effects, whereas, leaving resources unextracted has negative effect on the optimal value function.

Endogenous Contract Duration

Social Planner:

- The SP chooses both the terminal time T and the terminal stock x_T along with the extraction rate $q(t)$. The problem becomes:

$$\max_{q(t),\, T,\, x_T} \int_0^T e^{-vt} \left\{ pq(t) - c\left[q(t)\right] - D\left[q(t), l\right] - E\left[q(t)\right] \right\} dt + \varphi\left(x_T, T\right)$$

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Private Extraction Firm

- The private extractor's problem for an endogenous contract length becomes:

$$\max_{q(t),T,x_T} \int_0^T e^{-vt}\left\{pq(t) - c\left[q(t)\right] - r(\bar{l}) - R\left[q(t)\right]\right\} dt$$
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- For the private extraction firm problem:

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Results:

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- Comparative dynamics results are the same as those are in exogenous contract length case.

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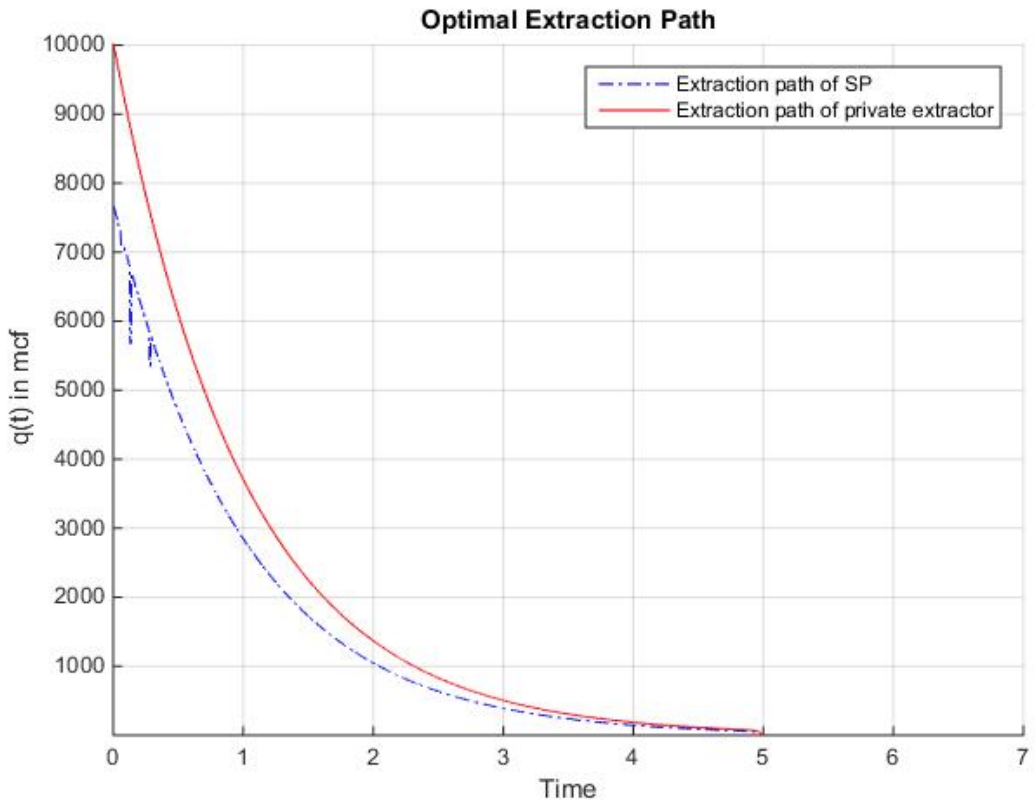
- Under reasonable assumptions the SP extracts at a lower rate compared to that of the private extractor.
- If the SP makes the private extractor to keep resources in situ, the optimal contract length will be longer.

Preliminary Simulation: Exogenous Contract Duration



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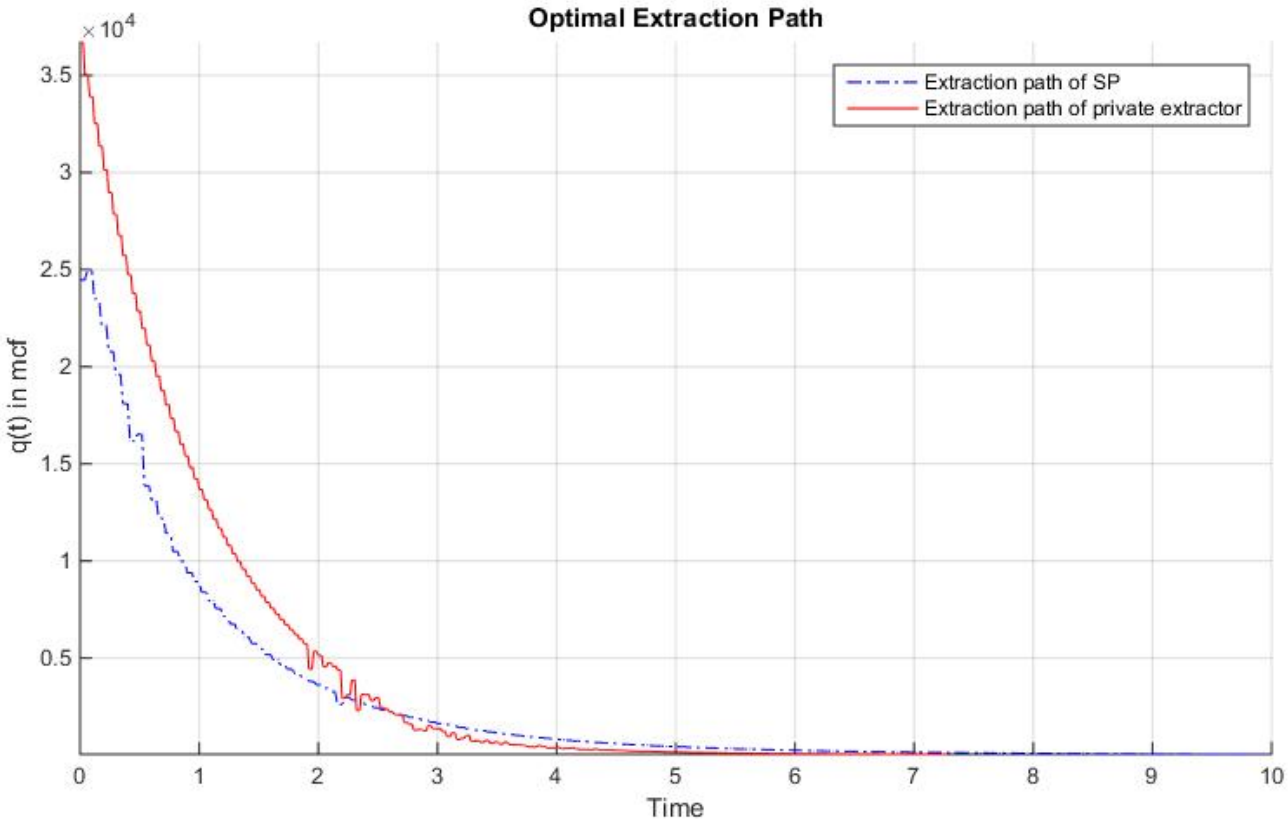

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- We can infer about extractor's behavior across different states incorporating government's preference towards this technology.
- The SP may have weights for the extractor's rent vs. the external damages.
- Suppose $\omega \in [0, 1]$ and $(1 - \omega)$ indicate the weight regulator assigns to external damages and rent from extraction respectively.

Potential Policy Instrument:

- The private extractor's problem from the regulator's perspective becomes:

$$\max_{q(t),T,x_T} \int_0^T e^{-vt}\{(1-\omega)[pq(t)-c[q(t)]-r(I)-R[q(t)]]-\omega E[q(t)]\}$$
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- The regulator can use a severance tax to reduce social cost.

Potential Policy Instrument:

- If τ is the tax rate on extraction we can derive

$$\tau^* = p - c'\left[q(t)\right] - R'[q(t)] - \frac{\omega}{(1-\omega)} E_q\left[q(t)\right].$$

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- This implies if the SP values the external costs more relative to extraction rent, optimal tax will be smaller.

Concluding Remarks

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- Comparison of social and private decisions can inform future policy concerning contract length and environmental damage in hydraulic fracturing applications.
- In our future work we will try to explore different regulations that can reduce social cost resulting from inefficient contract duration and extraction rate in hydraulic fracturing.