How Gasoline Prices Impact Household Driving and Auto Purchasing Decisions

A Revealed Preference Approach

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June 2010

Policy Relevance

- Accurately measuring the elasticity of demand for gasoline
  - Importance in climate policy models
  - Economic incidence of gasoline tax - burden falls on consumers or producers

Elasticity of Demand for Gasoline

- Prior studies have found very low elasticities
- Downward bias due to:
  - Assumptions
  - Research Methods

Model Objectives and Innovations

Incorporate the following into measurement of gasoline demand elasticity:

- *Extensive and Intensive* Margin (vehicle purchase decision and VMT)
  - Type of vehicle $\leftrightarrow$ Amount of driving
  - Estimate jointly:
    - Model choice
    - Fleet size
    - Driving demand

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    - Model choice
    - Fleet size
    - Driving demand
- Household fleet's VMT decisions *jointly determined*
  - Allocation of VMT between vehicles
  - Substitution as relative operating costs change

Model Objectives and Innovations

- *Vehicle Fixed Effects*
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- *Detailed* choice set
  - To capture subtle changes in vehicle purchase decision

Methodological Hurdle

- *High dimensionality* of choice set
- 2418 types of vehicles (model-year) in dataset
  - If households can choose 2: 2,922,153 possible choices
  - If households can choose 3: 2,353,307,216 possible choices
- Infeasible size of choice set for logit, probit models

Proposed Method

- Revealed preference approach:
  - Observed household vehicle holdings is equilibrium, provides maximum utility
  - Any deviation from equilibrium results in lower utility
    - Thus, can compare the utility levels:

Utility(observed) > Utility(deviation)

- Allows for unconstrained choice set and fixed effects

Literature Review

- Separate estimation for extensive/intensive margins
  - West (2007), Klier and Linn (2008)
- Joint margin estimation
  - West (2004), Goldberg (1998), Berkowitz et al. (1990)
- One step approach
  - Feng, Fullerton, and Gan (2005), Bento et al. (2008)
- Fleet model
  - Green and Hu (1985)

Model

Per-vehicle sub-utility:

$$u_{ij} = \alpha_{ij} VMT_{ij}^{\rho} + \gamma_i X_j + \xi_j + \varepsilon_{ij}$$

i: household

j: vehicle

VMT_{ij} = Vehicle Miles Travelled per year

X_j : observable attributes of vehicle j

ξ_j : unobservable attributes of vehicle j

$\varepsilon_{ij} \sim N(0, \sigma^2)$

Model

Marginal utility of driving:

$$\alpha_{ij} = \left(\overline{Z_i^{\phi}} A \overline{X_j^{\phi}} \right)^{1/\phi} = \left(a_1 Z_{i1}^{\phi} X_{j1}^{\phi} + a_2 Z_{i2}^{\phi} X_{j2}^{\phi} + \ldots a_t Z_{it}^{\phi} X_{jt}^{\phi} \right)^{1/\phi}$$

Z_i : Household i's attributes

Fixed effects:

$$\gamma_i = g_0 + g_1 Z_i$$

$$\theta_j = g_0 X_j + \xi_j$$

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Thus:

$$u_{ij} = \left(\overline{Z_i^{\phi}} A \overline{X_j^{\phi}} \right)^{1/\phi} VMT_{ij}^{\rho} + g_1 Z_i X_j + \theta_j + \varepsilon_{ij}$$

Model: Utility Maximization

$$\max_{VMT,c} \quad U_i = \sum_j u_{ij} + c_{ij}^{\rho}$$

$$s.t. \quad y_i = \sum_j P_j + \sum_j P_{ij}^d VMT_{ij} + P^c c_i$$

y_i : household income

P_j : vehicle *j*'s used price (opportunity cost of not selling)

P_{ij}^d : operating cost ($/mile)

P^c : price of consumption = 1

Interdependence of vehicles in fleet:

$$VMT_{ij}^{*} = \left(\frac{P_{ij}^{d}}{\alpha_{ij}}\right)^{\frac{1}{\rho-1}} \frac{\left(y_i - \sum_j P_j\right)}{\left(1 + \sum_j \left\{ \left(P_{ij}^{d}\right)^{\left(\frac{\rho}{\rho-1}\right)} \left(\frac{1}{\alpha_{ij}}\right)^{\frac{1}{\rho-1}} \right\}\right)}$$

→ Indirect Utility

$$V_i^{*} = U_i\left(VMT_{ij}^{*}, c_i^{*}\right)$$

First Stage Estimation: Swapping

- *Assumption 1*: Household in equilibrium with vehicle purchase and VMT decision

$$V_{iF_i^*} \geq V_{iF_i} \qquad \forall F_i \neq F_i^*$$

F_i^*: Fleet chosen by household i

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- Two households, 1 and 2, have vehicles A, B respectively:

$$\tilde{V}_{1A} + \theta_A \geq \tilde{V}_{1B} + \theta_B$$
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- Thus:

$$\tilde{V}_{1A} - \tilde{V}_{1B} + \tilde{V}_{2B} - \tilde{V}_{2A} \geq 0$$

First Stage Estimation

$$\Pr\left(\widetilde{V}_{1A} - \widetilde{V}_{1B} + \widetilde{V}_{2B} - \widetilde{V}_{2A} \geq 0\right) = \Pr\left(\widetilde{\varepsilon}_{12,AB} \leq \overline{V}_{1A} - \overline{V}_{1B} + \overline{V}_{2B} - \overline{V}_{2A}\right)$$

$$= \Phi\left(\frac{\overline{V}_{1A} - \overline{V}_{1B} + \overline{V}_{2B} - \overline{V}_{2A}}{\sigma_{\widetilde{\varepsilon}}}\right)$$

Maximum Likelihood:

$$LL(\beta) = \sum_{swaps_{(i_1,i_2)(j_1,j_2)}} \log \Phi\left(\frac{\overline{V}_{i_1 j_1} - \overline{V}_{i_1 j_2} + \overline{V}_{i_2 j_2} - \overline{V}_{i_2 j_1}}{\sigma_{\widetilde{\varepsilon}}}\right)$$

Normalization:

$$\beta_c = 1$$

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-Calculate objective function (summed log of differences)

-Find β' that increases objective function

-Repeat until convergence

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- Rewriting:

$$\bar{V}_{1A} + \theta_A + \varepsilon_{1A} \geq 0$$

$$\bar{V}_{2B} + \theta_B + \varepsilon_{2B} \geq \bar{V}_{2B,A} + \theta_A + \theta_B + \varepsilon_{2B} + \varepsilon_{2A}$$

Second Stage Estimation

- For Household 1:

$$P\left(\varepsilon_{1A} \leq \overline{V}_{1A} + \theta_A\right) = \Phi\left(\frac{\overline{V}_{1A} + \theta_A}{\sigma_{\varepsilon_A}}\right)$$

- For Household 2:

$$P\left(\varepsilon_{2A} \leq \overline{V}_{2B} - \overline{V}_{2A} - \theta_A\right) = \Phi\left(\frac{\Delta\overline{V}_2 - \theta_A}{\sigma_{\varepsilon_A}}\right)$$

Second Stage Estimation

- Maximum Likelihood:

$$LL(\theta_j) = \sum_i y_{ij} \ln \Phi\left(\frac{\Delta\bar{V}_i + \theta_j}{\sigma_{\varepsilon_j}}\right) + (1 - y_{ij}) \ln\left(1 - \Phi\left(\frac{\Delta\bar{V}_i - \theta_j}{\sigma_{\varepsilon_j}}\right)\right)$$

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- OLS:

$$\theta_j = g_0 X_j + \xi_j$$

Second Stage Estimation: Overview

- For each type of vehicle:
  1) Find all households who own it: positive sub-utility from owning it.
  2) Find all households who don’t own it: adding this vehicle decreases utility.
  3) Form maximum likelihood over (1) and (2)
  4) Estimate fixed effect for this vehicle
- Run OLS of all FE on vehicle characteristics

Data

- Household level data: NHTS 2001
  - National Sample:
    - 26,038 households
    - 53,275 observations
  - Final Sample:
    - 11,354 households
    - 18,166 observations

Data: NHTS Summary Statistics

	National Sample	Final Sample
% White	87%	85%
% Urban	70%	79%
Average Family Income	$55,832	$56,338
Average Household Size	2.82	2.66
Average Workers to Vehicles	0.65	0.72
Average Fleet Size	2.69	2.04
Average MPG	25.72	25.87
Average Vehicle Age (years)	8.49	7.21
Average Yearly VMT	10,995	11,594

Data: Vehicles

- Vehicle characteristic data: Polk, Ward's Automotive Yearbook
  - Provides detailed information on 18,273 vehicles 1971-2006
- Used vehicle prices: NADA
  - Provides used prices of all vehicles in 2001

Data: Gas

- American Chamber of Commerce Research Association (ACCRA) 2001 data
  - provides gas prices at the city level
  - yearly averages
  - aggregate to MSA

MSA	Gas Prices
Kansas City, KS	1.28
Houston, TX	1.34
Raleigh, NC	1.39
Chicago, IL	1.50
Philadelphia, PA	1.56
San Francisco, CA	1.92

Results: First Stage

	Interaction Term	Parameter Value (Std. Err.)
Marginal Utility of Driving Parameters	HP * urban	-2.5815*** (0.0203)
	Vehicle Size * household size	0.3689*** (0.0109)
	MPG * income	-0.0674*** (0.0425)
	ϕ : CES parameter (between Z_i and X_j)	1.0032*** (0.0000)
Indirect Utility of Driving Parameters	HP*income	0.6565*** (0.0619)
	MPG * urban	1.3418*** (0.1184)
	MPG * income	-0.2808*** (0.1322)
	Wheelbase * household size	2.7309*** (0.1785)
	ρ : CES parameter (between VMT and consumption)	0.4080*** (0.1968)
Variance of Error Term	$\sigma_{\tilde{\varepsilon}}$	2.8174*** (0.0254)
***: significant at 1% level		

Results

- Medium-run elasticity: allow for intra-fleet substitution
  - Parameter values imply: -1.0342*** (0.2692)

Conclusion

- Demand for gasoline is elastic
- Household choices are better represented
  - Discrete-continuous household portfolio model
  - Estimation method does not artificially restrict choice set

Any Questions?

Results: Second Stage Estimation

- OLS Regression:

$$\theta_j = g_0 X_j + \xi_j$$

	Parameter Value (Std. Err.)
Constant	18.166*** (3.5366)
Horsepower	-9.8413*** (0.4301)
MPG	3.4244***(0.2173)
Wheelbase	-0.9120 (2.7298)

Indirect Utility Function

- *Assumptions:*
  - Additive separability in θ_j and ε_{ij}
  - Non-linear in VMT_{ij}^*
  - Non-separable in VMT_{ij}^*
  - Composite error term $\tilde{\varepsilon}_{ij} = \sum_j \varepsilon_{ij} \sim N\left(0, \sigma_{\tilde{\varepsilon}}^2\right)$